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Box Inverse Modelling with
DOBOX 4.2

Phillip P. Morgan

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Phillip P. Morgan

CSIRO Division of Oceanography
GPO Box 1538, Hobart
Tasmania 7001
Australia

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Preface

Scope of Document

This document describes the methods involved in box inverse modelling as used by oceanographers to determine oceanic circulation, and the application of these methods using the computer software DOBOX version 4.2. It is assumed that the reader is familiar with physical oceanography, and is only a novice at computing. Chapters 2 and 3 are not written in the third person in an attempt to make the text clearer.

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1 Introduction

Carl Wunsch (1977) first introduced the technique of box inverse methods to physical oceanography. These methods involve partitioning an ocean into regions, enclosed (or "boxed in") by hydrographic sections and land masses. The property data (e.g. mass, salt, heat) and geostrophic velocities (relative to a predefined reference level) on a hydrographic section define property fluxes along each side of the boxes. A set of conservation equations for the flux of each property can be defined, and an inverse solution to this set of equations gives the estimated velocities at the defined reference level. Thus, the primary objective of using the box inverse method is to determine an estimate for the absolute velocities and property transports through the sides of all boxes and hence provide information on the general circulation of the ocean.

In the past, these methods have often relied on new models and new code being developed for each application. An improvement in productivity and consistency in solving such problems can be achieved by the development of a generalised box inverse modelling software package.

This report describes DOBOX, a software package to construct, invert and analyse box models. It was primarily designed for the determination of the general circulation of the oceans by inverse methods from hydrographic section data and the estimation of the transport of mass, salt, heat and other properties through such sections. DOBOX is written in the MATLAB language and can be used on any computer platform that supports MATLAB. No recompiling or conversion is necessary.

1.1 Formulation of Box Model

The theory and mathematics of the Box Inverse Model have been well discussed by Wunsch (1977, 1978). Only a brief outline of the theory will be presented here, though differences in notation will be highlighted.

Each boxed region of ocean can be divided vertically into several layers defined by the sea surface, the sea floor and K level surfaces. These level surfaces may be equipotential surfaces, isopycnal surfaces or some other surfaces that define layers in which we wish to conserve properties such as mass, salt and heat. The conservation equation for a property i in a layer k summed over J hydrographic station pairs is given by,

$$\sum_{j=1}^{J} \Delta d_j \delta_j \int_{P_k}^{P_{k-1}} C_{ij}(v_j + x_j)\,\mathrm{d}p = 0 \tag{1}$$

where Δd_j is the distance between adjacent hydrographic stations, C_{ij} is the property value per unit volume (property "concentration"), v_j is geostrophic velocity relative to a reference level, x_j is the unknown velocity at the reference level and P_k, P_{k-1} are the level surface pressures defining the kth layer. The unit normal $\delta = 1$ is chosen so that a positive value represents flow into the layer. Equation (1) can be rewritten as,

$$\sum_{j=1}^{J} \Delta d_j \delta_j \int_{P_k}^{P_{k-1}} C_{ij} dp \; x_j = -\sum_{j=1}^{J} \Delta d_j \delta_j \int_{P_k}^{P_{k-1}} C_{ij} v_j dp \tag{2}$$

where the left hand side is the transport by the barotropic (or reference) flow and the right hand side is the baroclinic (or relative) transport. If we write one such equation for each property in each of the K+1 layers, we have a set of K+1 simultaneous equations. An additional equation can be derived by adding the K+1 layers and thus representing the total transport of a property through the sections. These equations can be written in the matrix form,

$$\mathbf{A}^{B} x^{B} = b^{B} \tag{3}$$

where

$$\begin{aligned} A_{ij}^{B} &= \Delta d_j \delta_j \int_{P_k}^{P_{k-1}} C_{ij} dp \\ b_j^{B} &= -\sum_{j=1}^{J} \Delta d_j \delta_j \int_{P_k}^{P_{k-1}} C_{ij} v_j dp. \end{aligned} \tag{4}$$

Equation (2) only accounts for the lateral flux along a layer and through the sides of the box. To include cross-surface flow between layers (often thought of as a "vertical" flow), additional terms need to be added to the left hand side of equation (2) to give,

$$\sum_{j=1}^{J} \Delta d_j \delta_j \int_{p_k}^{p_{k-1}} C_{ij} dp \; x_j + \delta_j [(aCw)_k - (aCw)_{k-1}] = -\sum_{j=1}^{J} \Delta d_j \delta_j \int_{p_k}^{p_{k-1}} C_{ij} v_j dp \qquad (5)$$

where w_k, w_{k-1} are the interfacial velocities and a_k, a_{k-1} are the interfacial areas of the k and k-1 level surfaces bounding layer k. We can write equation (5) in the matrix form as,

$$\begin{bmatrix} \mathrm{A}^{B} & \mathrm{A}^{C} \end{bmatrix} \begin{bmatrix} x^{B} \\ w \end{bmatrix} = b^{B} \qquad (6)$$

where $A^{C}_{ij} = (aCw)_k - (aCw)_{k-1}$.

This system of equations can be augmented with additional constraint equations. These additional constraints can be appended to (5) to form the linear system,

$$\begin{bmatrix} \mathrm{A}^{B} & \mathrm{A}^{C} \\ \mathrm{A}^{R} & 0 \end{bmatrix} \begin{bmatrix} x^{B} \\ w \end{bmatrix} = \begin{bmatrix} b^{B} \\ b^{R} \end{bmatrix} \qquad (7)$$

$$\mathbf{A}\, x = b$$

where A^R and b^R are the reference and relative flow components of the extra constraints. The superscripts in equation (7) are chosen such that B represents the "base" set of equations (3) whereas R and C refer to the extra rows and extra columns that can be appended to the base system of equations.

1.2 Weighting the System

The mxn matrix A_{mxn} in (7) represents a system of equations where m is the number of equations and n is the number of unknowns. The relative emphasis on the m equations in (7) can be modified by weighting the rows of A_{mxn}. Similarly, the relative emphasis on the n unknowns can be modified by weighting the columns of A_{mxn}. If W^{R}_{mxm} and W^{C}_{nxn} denote row and column weighting matrices respectively, then equation (7) can be transformed into the weighted system,

$$\begin{aligned}(W^R A)\, x &= W^R b \\ A' x &= b'\end{aligned} \tag{8}$$

$$\begin{aligned}A' W^C (W^C)^{-1} x &= b' \\ A'' x' &= b'.\end{aligned} \tag{9}$$

1.3 Solving the System

Equation (9) is generally an underdetermined mxn system where there are less equations than unknowns (m<n). The system can be solved (Wunsch 1978, Lanczos 1961) by using the Singular Value Decomposition (SVD) technique. The matrix A'' can be decomposed into the SVD form,

$$A''_{mxn} = U_{mxk}\, S_{kxk}\, V^T_{kxn} \qquad (k \leq m < n). \tag{10}$$

where the matrices U and V contain orthonormal column vectors, S is a diagonal matrix of the singular values of A, and k is the rank of A. The solution to (7), (8) or (9) is obtained by a left-multiplication by the Moore-Penrose inverse ($VS^{-1}U^T$),

$$\begin{aligned}\hat{x}'_{nx1} &= (V_{nxk}\, S^{-1}_{kxk}\, U^T_{kxm})\, b'_{mx1} \\ \hat{x}_{nx1} &= W^C_{NxN}\, \hat{x}'_{nx1}\end{aligned} \tag{11}$$

1.4 Diagnostics

How well the solution resolves the unknowns is given by the model resolution matrix VV^T in the equation,

$$\hat{x} = (VV^T)\, x_{true}. \tag{12}$$

Similarly, the resolution of the equation is given by the equation resolution matrix UU^T in the equation,

$$\hat{b} = (UU^T)\, b_{true}. \tag{13}$$

These resolution matrices help in diagnosing the results of any solution. For example, the diagonal elements of UU^T give the relative information content of the equations (Wunsch 1978, Roemmich 1980).

2 Overview

DOBOX constructs the property equations in the general linear form, $Ax=b$, and inverts the system of equations to find the solution x, which gives geostrophic velocities at the reference level at each station pair. If the model has been formulated with cross-surface flow then the solution will also contain the interfacial velocities. The solution can be used to calculate the estimated property transports through the sections. The sign convention for the transports is that a *positive* sign is transport *into* a box.

Before using DOBOX, prepare "raw" section files in the DOBOX format (see Appendix 1) by some other application program. These section files contain the property fields, geostrophic velocities, station positions and the depth of the level surfaces at each station. The general procedure in using DOBOX involves carrying out the following steps:

1. Instruct DOBOX to calculate layer thicknesses, mean property values in each layer and various other parameters for a "raw" section file. The results are stored in a "processed" section file.

2. Instruct DOBOX how to construct the boxes (where properties will be conserved in each layer) from the individual "processed" section files. See Appendix 3 for details of the "geometry" file required.

3. Calculate the relative geostrophic velocities and transports in each layer relative to a defined reference level. See Appendix 2 for more information on how to calculate geostrophic velocities.

4. Define any extra rows and extra columns (Section 1.1), then construct the linerar system $Ax=b$. See Appendix 4 for details on the required formats for this extra information.

5. Define the row and column weights. DOBOX has several predefined weighting methods, but it also allows importing of row and column weights from other files.

6. Invert the set of equations and calculate the resultant transports.

7. Analyse the results using DOBOX or other tools.

The rest of this chapter describes how to start the DOBOX software, whereas Chapter 3 describes the above steps in detail using the various menus and options within DOBOX.

2.1 Getting Started

Start MATLAB and type the command **dobox** which will display an announcement screen. Press a key to dismiss the announcement and the main menu for DOBOX will be displayed as shown below.

```
>> dobox
 ==============================================================
                                DOBOX 4.2

      Box Inversion Package for Oceanographic Research at CSIRO


                                   by

                                Phil Morgan


       Please send bug reports to morgan@ml.csiro.au
          (Include a copy of your MATLAB session)
 ==============================================================

Hit a key to continue...


DOBOX Release 4.2

----- BOX MENU (0) : Model Choices -----

      1) Prepare       -    Raw Section file for DOBOX
      2) Lhs           -    Define geometry, Build Abase
      3) Rhs ...       -    Set reflevel, Build bbase
      4) Build ...     -    Define extra Row/Columns, Build A & b
      5) Weight ...    -    Define Row/Column weighting
      6) Solve ...     -    Solve Ax=b, evaluate transports
      7) Analyse ...   -    Analysis plots

      0) Matlab        -    Return to MATLAB


  Select a menu number: {2} 2
```

The main menu displays a list of menu numbers associated with an action to be performed and a brief description of each action. Actions that have a trailing ellipse (...) indicate that the action involves a sub-menu.

The prompt (`Select a menu number`) asks you to enter a menu item by number. The number in braces is DOBOX's best guess at the next menu item, however, you must explicitly enter a number. You must choose the options from the main menu in ascending order. If you wish to go back and reset a previous option then you must continue new options from there in ascending order.

2.2 Convention for Variable Names

The variables in equations (1) to (11) are given descriptive variable names in DOBOX and are used throughout the following discussion. The convention used is described in the following table.

Symbol	Descriptive Variable Name
A^B	Abase
A^C	Xcols
A^R	Xrows
b^B	bbase
b^R	Xbs

3 Menus

A brief overview of each of the options listed in the main menu is described in the following sub sections.

3.1 Prepare - Raw Section file for DOBOX

For each section, DOBOX requires you to supply a data file containing the property fields, geostrophic velocities, station positions and the depth of the surfaces at each station. The "Prepare" option (action #1) identifies layer boundaries, calculates layer thickness and calculates the mean property concentrations in layers for each raw section file. This option is generally only carried out once and the processed data saved to a new file. Such prepared section files will be named in the geometry definition file that is discussed below.

Full details of the data format required for the raw section files is found in Appendix 1. You should ensure that your raw section files only contain the variable names as required since extraneous data may conflict with the internal operations of DOBOX.

3.2 Lhs - Define geometry, Build Abase

DOBOX requires a geometry definition file that defines:

- the section file names by the variable **sectfiles**,
- the names of properties to be conserved by the variable **properties**,
- how the boxes are to be constructed from the sections by the variable **geometry**.

Full details on how to create the geometry definition file are given in Section 4. DOBOX uses these definitions to construct the matrix A^B (called "Abase" in DOBOX), on the left hand side of the equation (3), for the basic system comprising only the lateral flow. The matrix Abase is made up of area times property concentration for each layer at each station pair.

Selecting option 2 from the main menu prompts you to enter the file name of the geometry definition file. This geometry file must be in the current working directory.

3.3 Rhs ... - Set reflevel, Build bbase

This option constructs the right hand side (bbase) of equation (3) from the geostrophic velocities relative to the reference level and property concentrations. The sub-menu presented is,

```
----- RHS MENU (3) -----

      1) Set Reference level - build TL
      2) Reset Ekman to zero
      3) Calculate Ekman

      0) Main Menu


  Select a menu number: 1

 What reflevel (-1=dcom depth, 0=surface, +n=level) ? 5
```

The reference level is specified via sub-menu option 1 as: the deepest common depth at the each station pair (enter -1), the sea surface (enter 0), or a particular level surface (enter the surface number).

You also have the option to add the contribution of the Ekman transport across each section using the annual mean winds. By default, DOBOX uses the wind climatology derived from the ECMWF analysis by Trenberth (et al. 1989). The resultant lateral transports into a box, relative to the reference level, will have a positive sign.

3.4 Build ... - Define extra Row/Columns, Build A & b

Selecting option 4 from the main menu will display the following sub-menu:

```
----- BUILD MENU (4) : Extra rows/columns and Build A & b -----


      1) No Xrows or Xcols

      2) Load Xrows from file
      3) Load Xcols from file

      0) Main Menu  [Builds A & b]


  Select a menu number: 0
```

You must select an action from this sub-menu. Extra rows (e.g. to include extra equations) and extra columns (e.g. to include extra unknowns, such as interfacial fluxes) may be added by selecting options 2 or 3 respectively. If either of these two options are chosen, block matrices Xrows, Xcols and Xbs will be appended to A^B and b^B as described by equations (7) and (14).

$$\begin{bmatrix} Abase & Xcols \\ Xrows & Xzeros \end{bmatrix} x = \begin{bmatrix} bbase \\ Xbs \end{bmatrix} \tag{14}$$

If option 2 is selected, you are prompted to enter the name of a data file that contains Xrows, Xbs and XRW. XRW are the row weights for the rows of Xrows. If option 3 is selected, you are prompted to enter the name of a data file that contains Xcols and XCW. XCW contains the column weights for Xcols. DOBOX constructs Xzeros automatically from Xrows and Xcols. See section 4.2 for details on the required format of the variables in these two files.

3.5 Weight ... - Define Row/Column weighting

The solution to the system of linear equations (7) can be modified by weighting the rows and columns as described by equations (8) and (9). DOBOX uses a matrix RW for the row weights and a matrix CW for the column weights of A. The descriptive form of equations (8) and (9) can be written as,

$$\begin{aligned} (RW\ \ A)\ \ x &= RW\ \ b \\ Adash\ \ x &= bdash \\ A'\ \ x &= b' \end{aligned} \tag{15}$$

$$\begin{aligned} (Adash\ \ CW)\ \ (CW^{-1}\ \ x) &= bdash \\ Adashdash\ \ xdash &= bdash \\ A''\ \ x' &= b' \end{aligned} \tag{16}$$

The weights sub-menu allows you to set the row and column weights separately:

```
----- WEIGHTS MENU (5) -----

      1) Row    Weights
      2) Column Weights

      0) BOX menu


  Select a menu number: 1
```

3.5.1 Row Weighting

The row weights matrix (RW) is constructed from component blocks as shown by,

$$\begin{aligned} RWbase &= RWrel \;.\!* \; RWmag \\ RW &= [RWbase;\; XRW] \end{aligned} \qquad (17)$$

where RWbase is the row weight matrix for Abase and XRW is the row weight matrix for Xrows (if defined). The sub-menu for constructing RW is:

```
----- ROWWEIGHT MENU (51) -----

  SET RELATIVE ROW WEIGHTS (RWrel):
      1) Set RWrel to ones
      2) Set RWrel from *.m file

  SET ROW WEIGHT (RWmag) METHODS:
      3) Ones - identity
      4) Normalise with layer property mean
      5) Enter user defined row weights (RWmag) from *.mat file

  RESET XRW:
      6) Reset XRW from *.m file (overrides BUILD menu)

      0) Return to WEIGHTS menu

  Select a menu number: 1
```

The magnitude of the weights for the Abase matrix are stored in the vector RWmag. To weight each equation by normalising with the layer property mean, select option 4. To set all equations with the same weight select option 1 or enter your own values via option 5.

Often one may wish to selectively down-weight certain equations relative to other equations. For example, one may wish to down-weight the equations for layers near the sea surface. Relative weighting is done by specifying values in RWrel. These values are usually in the range from zero (no contribution) to one (full contribution).

XRW contains the weights for Xrows and should have been defined in option 4 (Build ...) of the main menu. However, you may reset these weights, via option 5, by supplying the name of an mfile that contains XRW.

3.5.2 Column Weighting

The column weights matrix (CW) is constructed from component blocks as shown by,

$$\begin{aligned} CWbase &= CWrel\ .*\ CWmag \\ CW &= [CWbase;\ XCW]. \end{aligned} \tag{18}$$

where CWbase is the column weight matrix for Abase and XCW is the column weight matrix for Xcols (if defined). The sub-menu for constructing CW is:

```
----- COLUMN WEIGHT MENU (52) -----

  SET RELATIVE COLUMN WEIGHTS (CWrel):
      1) Set CWrel to ones
      2) Set CWrel from *.m file

  SET COLUMN WEIGHT (CWmag) METHODS:
      3) Ones - identity
      4) Normalise with Pair Area
      5) Enter column weights (CWmag) from *.mat file

  RESET XCW:
      6) Reset XCW from *.m file (overrides BUILD menu)

      0) Return to WEIGHTS menu

  Select a menu number: 1
```

All the column weight options work in a similar fashion to the row weight sub-menu. The only difference is in option 4, where you can weight each column by normalising with the station pair area (the area of the station pair from sea surface to the deepest common depth).

3.6 Solve ... - Solve Ax=b, Evaluate transports

The first step in obtaining a solution is to perform the SVD decomposition (equation 10) of matrix A. A Levenberg-Marquart plot (Lawson and Hanson 1974) can be displayed to aid in choosing an appropriate rank. After a rank is chosen, option 3 computes the solution (equation 11) and evaluates the property transports.

```
----- SVD MENU (61) : -----

      1) Decompose A
      2) Levenberg-Marquardt plot
      3) Set rank, solve for x, compute transports

      0) Main menu

  Select a menu number: 1
```

3.7 Analyse ... - Analysis plots

The analysis menu is shown below:

```
----- Analysis of Box results -----

      1) Model Resolution for section
      2) Equation Resolution for box
      3) x for section

      4) Section       Transport
      5) Net section   Transport
      6) Net box       Transport
      7) Ekman section Transport

      8) Print table of section fluxes to screen
      9) Keyboard - enter your command

     10) Box Lateral & Net transports

      0) Main Menu

  Select a menu number: {1} 1
```

Option 1 plots a line graph of the diagonal of the model resolution matrix (equation 12) for a specified section. If Xcols are included in the model then the model resolution for the unknown w (equation 7) can be plotted by specifying the section number as "nsections+1".

Option 2 plots the diagonal of the equation resolution matrix (equation 13) for a specified box. If Xrows are included then the resolution for the extra equations is obtained by specifying the box number as "nboxes+1".

Option 3 plots the reference level velocities for a specified section. The interfacial velocities can be plotted by specifying the section number as "nsections+1".

Options 4, 5, 6, 7 and 10 plot bar charts of the various transports viewed layer-wise and station-wise. Option 8 prints a table of the transports. Option 9 allows you to enter your own commands.

It is possible to add other routines into the analysis it is important to understand the construction of the $Ax=b$ system and the set of box/section/property data structure pointers. See Appendix 5 for details on data structure pointers.

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Appendix 1. Required Section File Formats

"Property fields" for hydrographic sections are the values of some property of sea water, placed on a common grid, and stored in matrices. Each column of these matrices contains property values for a station. The rows correspond to property values at discrete depths or averaged into depth bins. The property fields may also be properties from numerical models or bottle data at specified depths.

The variable **bin_press** specifies the depth of each bin for all stations. The gridded property fields must be named bin_PropertyName where PropertyName is the name of the property that is to be conserved. For example, **bin_sal** would refer to the property "sal" or salinity.

The positions of the level surfaces at each station is specified in the variable **surf_press**. The pressure of these level surfaces define the layers in which properties are to be conserved.

The following table lists all the data required to be in a raw section file. Bad or non-existing data should be flagged as NaN. All the required data and associated variable names are described below. Variables with the Flag specified as "none" should not have any "bad" data elements.

Required Variables for Raw Sections File

Variable	Description	Dimensions	Units	Flag
bin_prop	Value of property "prop" in each pressure bin at each station	nbins x nstations	"prop" units	NaN
bin_press	pressure at centre of bin	nbins x 1	db	none
binPair_vel	geostrophic velocity at station pair for each bin	nbins x npairs	m/s	NaN
surf_press	pressure of each level-surface at each station	nsurfs x nstations	db	NaN
lat	latitude	1 x nstations	-90 ... +90	none
lon	longitude	1 x nstations	180 ... +180	none

where: nstations = number of stations in section
nbins = maximum number of bins in section
npairs = number of station pairs (nstations-1)

There are a number of restrictions on the input data as follows:

1. All property fields must have valid data points at the same places
2. **binPair_vel** and the pair-wise average of each bin_prop must have valid data points at the same places
3. **binPair_vel** must be relative to the sea surface (see Appendix 2).
4. A section must be completely in one hemisphere.

Appendix 2. Geostrophic Velocities

The concept of geostrophic velocity is well covered by Pond and Pickard (1986). The geostrophic velocity at a level (V_1) relative to a lower level (V_2) between two stations denoted by A and B is estimated by

$$V = V_1 - V_2 = \frac{(\Delta\Phi_B - \Delta\Phi_A)}{2d\Omega Sin\phi_{ave}} \quad [\mathrm{m\ s^{-1}}],$$

where d is the distance between stations, ϕ_{ave} is the average latitude and Ω is the angular velocity of the Earth. The geopotential anomaly ($\Delta\Phi$) is the integral of the specific volume anomaly (δ),

$$\Delta\Phi = \int_{P_1}^{P_2} \delta\, dp \quad [\mathrm{J\ kg^{-1}}].$$

If $\Delta\Phi_B < \Delta\Phi_A$ (station B has less dense water), then V is positive. If we take a cross sectional view of the station pair with "A" on the left and "B" on the right then the positive velocity is directed into the paper (in the northern hemisphere). This is the convention "light is on the right"; velocity is positive and is directed into the paper. Thus, if we traverse the section from station "A" to "B", we go from dense to less dense (or cold to hot) and the positive velocity sign is directed to the left of the path. The alternate view is to travel from "B" to "A", less dense to dense (or hot to cold) and the positive velocity is to the right of the path. This last description is in the same sense as the wind direction in a high pressure system (anticyclonic) in the northern hemisphere, which we are all used to seeing on weather maps - wind travels normal to the pressure gradient and is directed to the right as we traverse down a pressure gradient. In the Southern Hemisphere the signs will be reversed.

It is further possible to project the velocity vector which is normal to the station pair onto the conventional x, y coordinates, for which East is directed along the x axis and North along the y axis. This additional transformation is not to be used for the geostrophic velocities in DOBOX.

Geostrophic velocity calculations can be verified by comparison with the example given by Pond and Pickard (1986). Their geostrophic velocity calculations, relative to the bottom, between station A (41°55'N, 50°09'W) and station B (41°28'N, 50°09'W) are reproduced in the following Table. DOBOX requires geostrophic velocities **relative to the sea surface** and are shown in the last column of the table.

	Station A		**Station B**		**gvel (m/s)**	**gvel (m/s)**
depth(m)	Temp	Sal	Temp	Sal	rel to bottom	rel to top
0	5.99	33.71	13.04	35.62	0.26	0
200	8.85	35.03	12.65	35.54	0.24	-0.02
1000	3.90	34.95	4.20	34.97	0	-0.26

The magnitude of the geostrophic velocities that you calculate may differ somewhat due to the approximate nature of the distance calculation. However, it is most important that the correct sign be calculated. Morgan (1994) has developed a software package called SEAWATER which can calculate geostrophic velocities.

Appendix 3. Describing Geometry of System

Hydrographic sections have the stations arranged in an order such that the cruise path along a section can be thought of as a vector. A box is bounded by an arbitrary number of sections and land masses. DOBOX requires knowledge of how the boxes are constructed. The method used here is to trace the outline of a box (sections) in a counter-clockwise direction. If a particular section points in a counter-clockwise direction then it scores +1, if it points in the opposite direction it scores -1, and if the section is not used in constructing the box then it scores zero. An incidence matrix named **geometry** is constructed to describe the geometry. The box number is aligned along the rows, whereas the section number is aligned along the columns. Enter -1, 0 or +1 for each section (column) in each box (row) according to the above method. A variable **sectfiles** specifies the file names for each of the hydrographic sections with the first file corresponding to section number 1.

An example will help to illustrate the method. A plan view of a model comprising 4 sections and 2 boxes is shown below. *Section numbers* are enclosed in parentheses, whereas the *box number* is enclosed by square brackets. Tracing the sections in box [1] in a counter-clockwise direction gives a score of -1 for section (1), a score of -1 for section (2), and a score of -1 for section (3). Traversing box [2] gives a score of +1 for section (3) and a score of +1 for section (4). A schematic of the boxes is shown as,

```
     (1)
---------->
.         |
.         |
.   [1]   | (2)     2 box model constructed from
.         |         4 sections.  Dots indicate
.   (3)   V         land masses enclosing box.
<---------.
.         .
.  [2]    .
.         .
.  (4)    .
--------->
```

The corresponding geometry file (a MATLABmfile) entries would be

```
geometry =   [  -1  -1  -1  0  ;
                 0   0   1  1 ]

sectfiles =  ['data/file1.mat';
              'data/file2.mat';
              'data/file3.mat';
              'data/file4.mat' ] ;

properties = ['mass';        % mandatory first property
              'sal ';        % salinity          psu
              'ptmp';        % potential temp    C
              'oxy '];       % dissolved oxygen ml/l
```

It is also necessary to define a matrix called **properties** that lists, by name, the properties in the section files that are to be used as constraints. Note that all rows of this matrix must have the same number of columns so it may be necessary to pad with blanks. The property names must match the names of properties in the section files and the first property must be named mass. The properties are referred to by the position number in the **properties** array. Thus 'mass' is property 1, 'sal' is property 2 and so on.

Appendix 4. Extra Rows and Extra Columns

Both the extra rows and the extra columns are optional. Section 3.5 showed how these are appended to the system of equations. The extra rows and extra columns can be incorporated into DOBOX by specifying MATLAB data files which contain the matrix variables described in the following Table where the size of the matrices are shown in parentheses:

Extra Rows File	Extra Columns File
Xrows (nr x nAbase) XRW (nr x 1) Xbs (nr x 1)	Xcols (mAbase x nc) XCW (1 x nc)

where:
nr = number of extra rows
nc = number of extra columns
mAbase = number of rows of Abase
nAbase = number of columns Abase

Appendix 5. Data Structure Pointers

There are a number of data structure pointers that may be of use to those who wish to add other analysis routines. A list of some variables that are of interest follow:

Data Elements

nboxes = Number of boxes
nsections = Number of sections
nproperties = Number of properties
nlayers = Number of layers
nsurfs = Number of surfaces
mAbase = Number of rows in Abase
nAbase = Number of columns in Abase
mA = Number of rows in A
nA = Number of columns in A

Data structures

TL = matrix(mAbase x nAbase)
reLative property Transports
Units: "prop"units * m^3/s Flag: 0

TF = matrix(mA x nA)
reFerence property Transports
TF(i,:) = A(i,:) .* x
Units: "prop"units * m^3/s Flag: 0

TA = matrix(mAbase x nAbase)
Absolute property Transports for basic problem
TA = TF(1:mAbase,1:nAbase) + TL + E
Units: "prop"units * m^3/s Flag: 0

E = matrix(mAbase x nAbase)
Ekman property transports
Units: "prop"units * m^3/s Flag: 0

Data Structure Index Pointers

Access to parts of the matrices A, E, TL, TF and TA is achieved by using the following arrays of index pointers:

col_beg = column number where each section begins
col_end = column number where each section ends
row_beg = row number where each box begins
row_end = row number where each box ends
eqn_beg = row offset where each property equation begins
eqn_end = row offset where each property equation ends

The equations for total transport of properties can be accessed by using the row index of eqn_end(nlayers+1).

Examples Using Data Structures

A few simple examples will help to see how to use these data structures and pointers.

Example 1: The block matrix for the absolute property flux for property number "iprop" in section number "isect" into box number "ibox" is shown below. This is option 4 in the analysis menu and clearly demonstrates how easy it is to use the pointers.

```
disp('Plots net property transport for a section of a box')
ibox  = input('What box number? ');
isect = input('What section number ? ');
iprop = input('What property number ? ');
row1  = row_beg(ibox) + eqn_beg(iprop);
row2  = row_beg(ibox) + eqn_end(iprop);
rows  = [ row1  : row2 ];
cols  = [ col_beg(isect) : col_end(isect) ];

TAsect = TA(rows,cols);
```

Thus plotting the transport 'station-wise' can be achieved by

```
bar( sum(TAsect));
```

and plotting the transport 'layer-wise' can be achieved by

```
bar( sum(TAsect'));
```

Example 2: The net transport of property number 1 in box number 1 can be evaluated as:

```
ibox  = 1
iprop = 1
cols = 1:col_end( nsections)
row1 = row_beg( ibox ) + eqn_beg( iprop )
row2 = row_beg( ibox ) + eqn_end( iprop )
rows = row1:row2
box1flux = sum( sum( TA(rows,cols) ) )
```

CSIRO Marine Laboratories

Division of Oceanography

Headquarters
Castray Esplanade, Hobart, Tasmania
GPO Box 1538, Hobart, Tasmania, 7001, Australia

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