An ensemble-based method for optimal array design

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Theory

What is an optimal observation?

An optimal observation is the observation that maximises the reduction of uncertainty in the system state.

$$H^{opt} = \min_{\{H\}} \; \|P^a\|,$$

$$P^a = \left[I - P^b H^T (HP^b H^T + R)^{-1} H \right] P^b$$

System too big to handle covariance P? Use ensemble A:

$$P \leftrightarrow \frac{1}{m-1} AA^T$$

Theorem: serial assimilation and parallel assimilation of non-correlated observations result in the same analysed covariance.

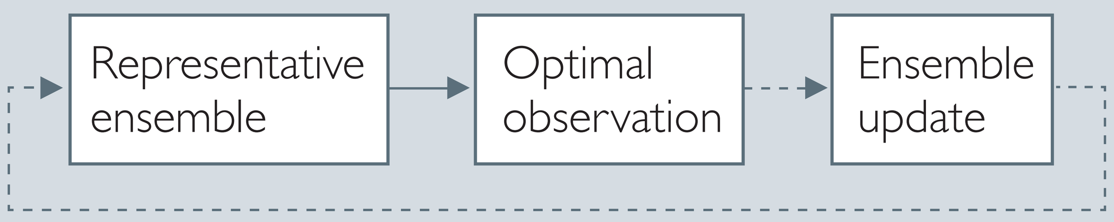

FIGURE 1: Serial calculation of an optimal observation array

Solution for optimal observation:

$$H^{opt} = \max_{\{H\}} \; \mathrm{tr} \; \frac{HA(A^T A)(HA)^T}{HA(HA)^T + (m-1)R_H}$$

Ensemble update:

$$A^a = A^f T, \quad T = \left(I + \frac{A^{f\,T} H^T R^{-1} H A^f}{m-1} \right)^{-1/2}$$

Models

Table 1: Summary of model configurations; FC refers to a flux correction.

	ACOM2	ACOM3	OFAM
Model code	MOM2	MOM3	MOM4
Zonal resolution	2°	0.5°	0.1-2°
Meridional resolution	0.5°-1.5°	0.33°	0.1-2°
Number of vertical levels	25	33	47
Wind forcing	NCEP/NCAR + FSU	ERS1/2	ERA40
Non-shortwave heat flux	ABLM + FC	ABLM + FC	ERA40 + FC
Shortwave heat flux	as above	OLR + NCEP	ERA40
Freshwater flux	as above	Monthly analyses	Levitus
Simulated period	1982-1994	1992-2000	1992-2005

Application: IMLD in TIO

IMLD = intraseasonal (<100 days) mixed layer depth.

Table 2: The basin-averaged theoretical analysis error variance of IMLD (m^2); and the percent reduction in parentheses.

	Ensemble		
	ACOM2	ACOM3	OFAM
Signal	15.2	22.7	49.3
Proposed array	12.3 (19%)	16.7 (26%)	36.6 (26%)
ACOM2 array	11.5 (24%)	16.2 (29%)	36.6 (28%)
ACOM3 array	11.9 (22%)	15.4 (32%)	35.8 (27%)
OFAM array	12.1 (20%)	16.3 (28%)	33.6 (32%)

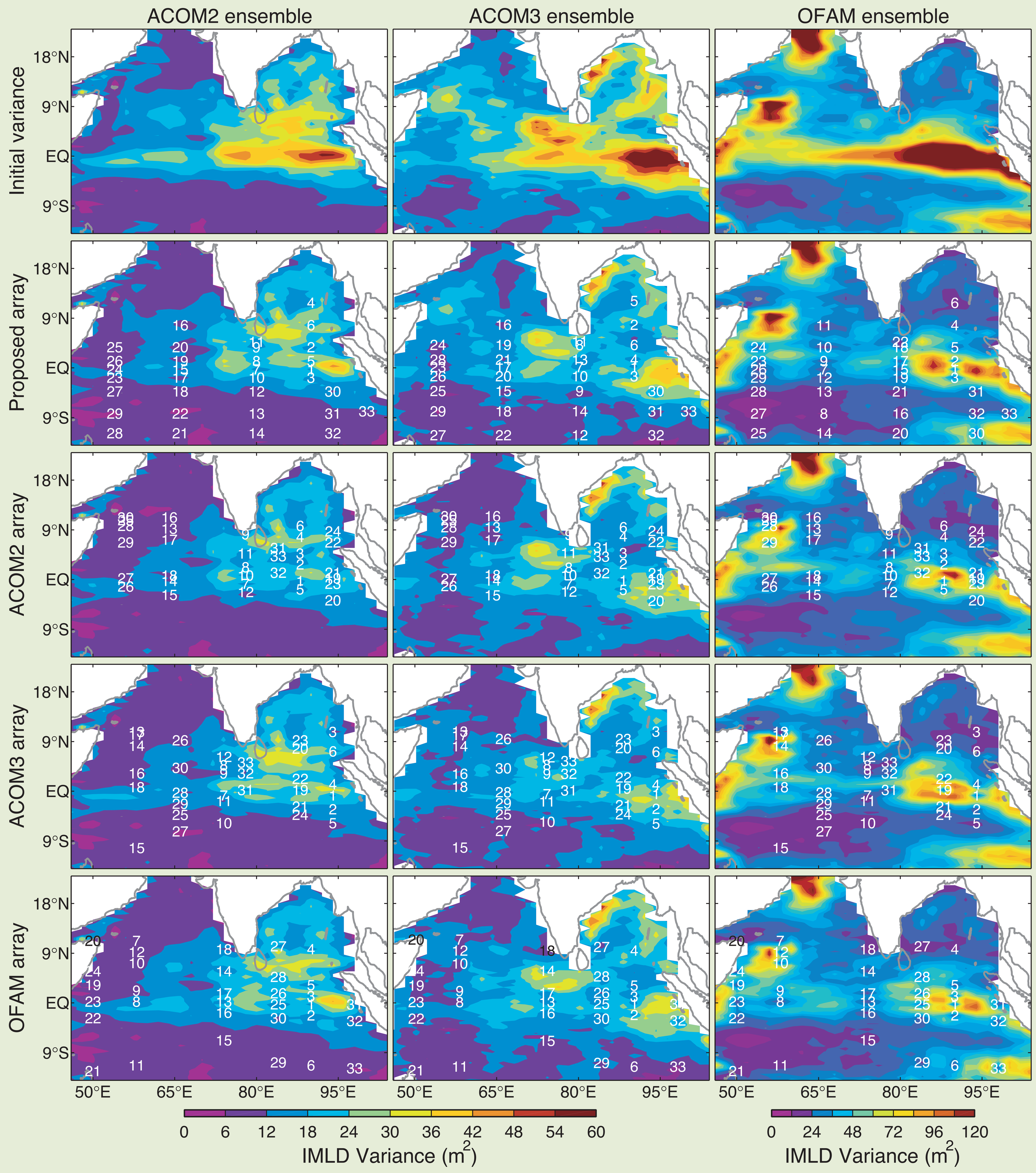

FIGURE 2: The variance of the IMLD signal variability (top) in ACOM2 (left), ACOM3 (center) and OFAM (right); and the theoretical analysis error variance for each model using the proposed, ACOM2-, ACOM3- and OFAM-derived arrays (top-bottom); as labeled to the left or each row. The numbers in each panel denote the mooring locations and the ranking of each location (i.e., the locations marker "1" are the best location).

Application: SLA in TIO

SLA = sea-level anomalies

Table 3: As for Table 2, except for experiments using observed SLA.

	Variance (cm²)
Signal	81.0
Proposed array	34.9 (57%)
Unstructured array	27.1 (66%)
Structured array	29.5 (64%)

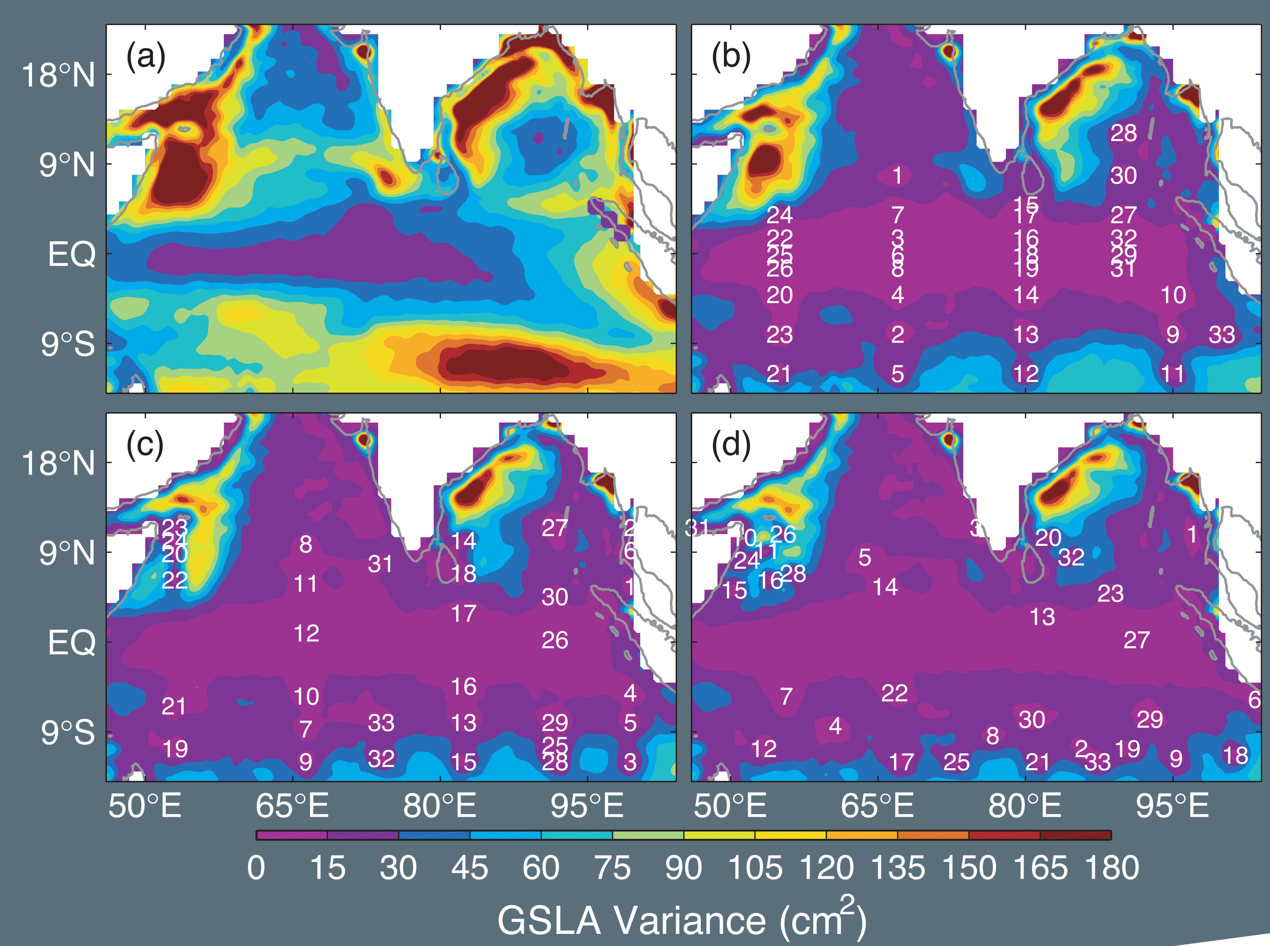

FIGURE 3 (right): (a) Variance of the observed SLA and the theoretical analysis error variance using (b) the proposed array, (c) a structured optimal array, and (d) an unstructured optimal array. The numbers denote both the mooring locations and the ranking of each location as in Figure 2.

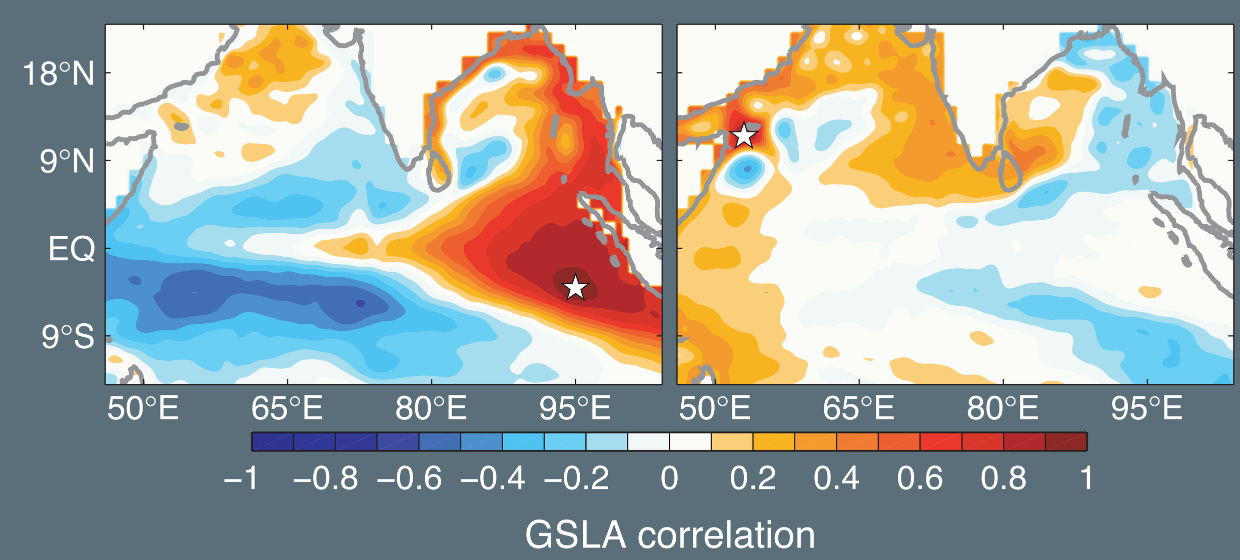

FIGURE 4: Correlations between SLA at a reference location, denoted by the star, and SLA elsewhere.

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