On the correct implementation of the ensemble square root Kalman filter

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We compare the performance of mean-preserving and non-mean-preserving solutions for the ensemble transform in the ensemble square root filter (ESRFs) and demonstrate a significant advantage of the former. We argue that only mean-preserving solutions should be used in practice.

Theory

• ESRF analysis scheme:

1 Given a forecast ensemble $\mathbf{X}^f$, calculate the ensemble mean $\mathbf{x}^f$ and the ensemble anomalies $\mathbf{A}^f$:

$$\mathbf{x}^f = \frac{1}{m}\sum_{i=1}^{m}\mathbf{X}_i^f, \quad \mathbf{A}_i^f = \mathbf{X}_i^f - \mathbf{x}.$$

2 Calculate the analysis $\mathbf{x}^a$:

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}(\mathbf{d} - \mathbf{H}\mathbf{x}^f),$$
$$\mathbf{K} = \mathbf{P}^f\mathbf{H}^T(\mathbf{H}\mathbf{P}^f\mathbf{H}^T + \mathbf{R})^{-1}.$$

3 Calculate the analysed anomalies:

$$\mathbf{A}^a = \mathbf{A}^f\mathbf{T},$$
$$\mathbf{T}: \quad \mathbf{A}^f\mathbf{T}(\mathbf{A}^f\mathbf{T})^T = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{A}^f\mathbf{A}^{fT}.$$

4 Calculate the analysed ensemble $\mathbf{x}^a$:

$$\mathbf{X}^a = \mathbf{A}^a + [\mathbf{x}^a, \ldots, \mathbf{x}^a].$$

• General mean-preserving solution for the ensemble transform:

$$\boxed{\mathbf{T} = \mathbf{T}^s\mathbf{U}^p}, \quad \mathbf{U}^p\mathbf{U}^{pT} = \mathbf{I}, \quad \mathbf{U}^p\mathbf{1} = \mathbf{1},$$

$$\mathbf{T}^s = \left(\mathbf{I} + \frac{\mathbf{A}^{fT}\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\mathbf{A}^f}{m-1}\right)^{-1/2} = \mathbf{C}\Gamma^{-1/2}\mathbf{C}^T,$$

$$\text{where} \quad \mathbf{C}\Gamma\mathbf{C}^T = \mathbf{I} + \frac{\mathbf{A}^{fT}\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\mathbf{A}^f}{m-1}.$$

• Some particular non-mean-preserving solutions used in practice:

one-sided solution $\mathbf{T} = \mathbf{C}\Gamma^{-1/2}$ (Tippett *et al.*, 2003)

solution with random rotations $\mathbf{T} = \mathbf{C}\Gamma^{-1/2}\mathbf{U}, \ \mathbf{U}\mathbf{U}^T = \mathbf{I}$ (Evensen, 2004)

Numerical experiments

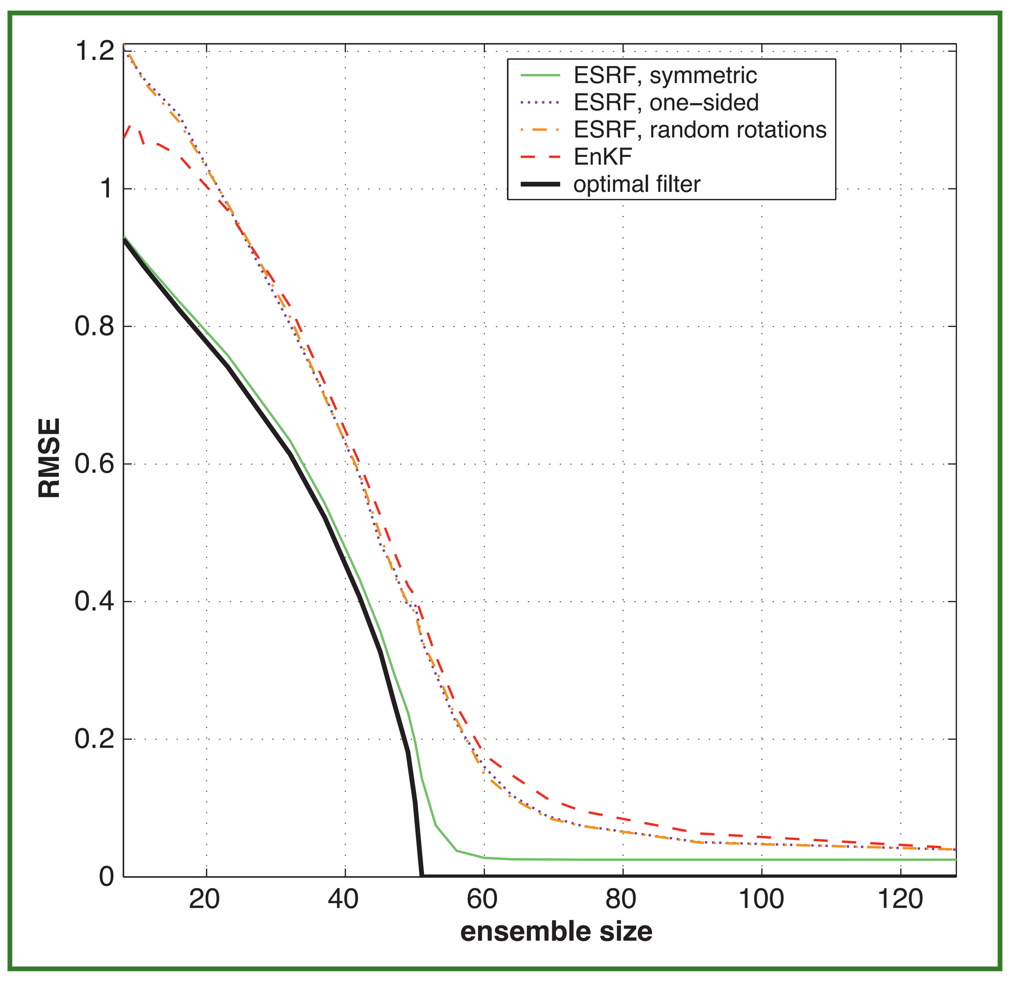
Models:

- Linear advection (LA) model (Evensen, 2004), Figures 1 and 2.
- Lorenz-40 model (Lorenz and Emanuel, 1998; setup as in Whitaker and Hamill, 2002), Figures 3 and 4.

Filters:

- EnKF
- ESRF, symmetric
- ESRF, one-sided
- ESRF, with random rotations
- ESRF, with mean-preserving random rotations

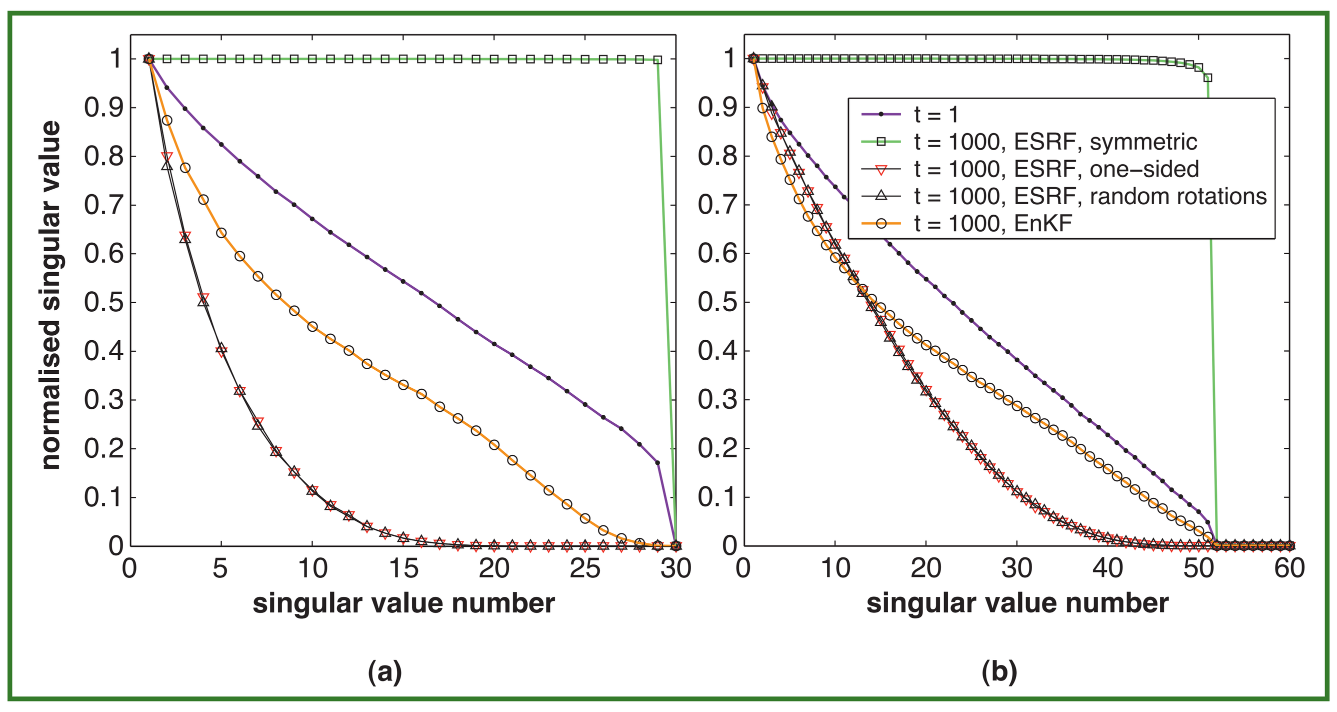
Figure 1



<<Figure 1

RMSE of different flavours of ESRF and EnKF for the LA model, averaged for the time interval t = [900, 1000], and over 50 realisations.

Figure 2



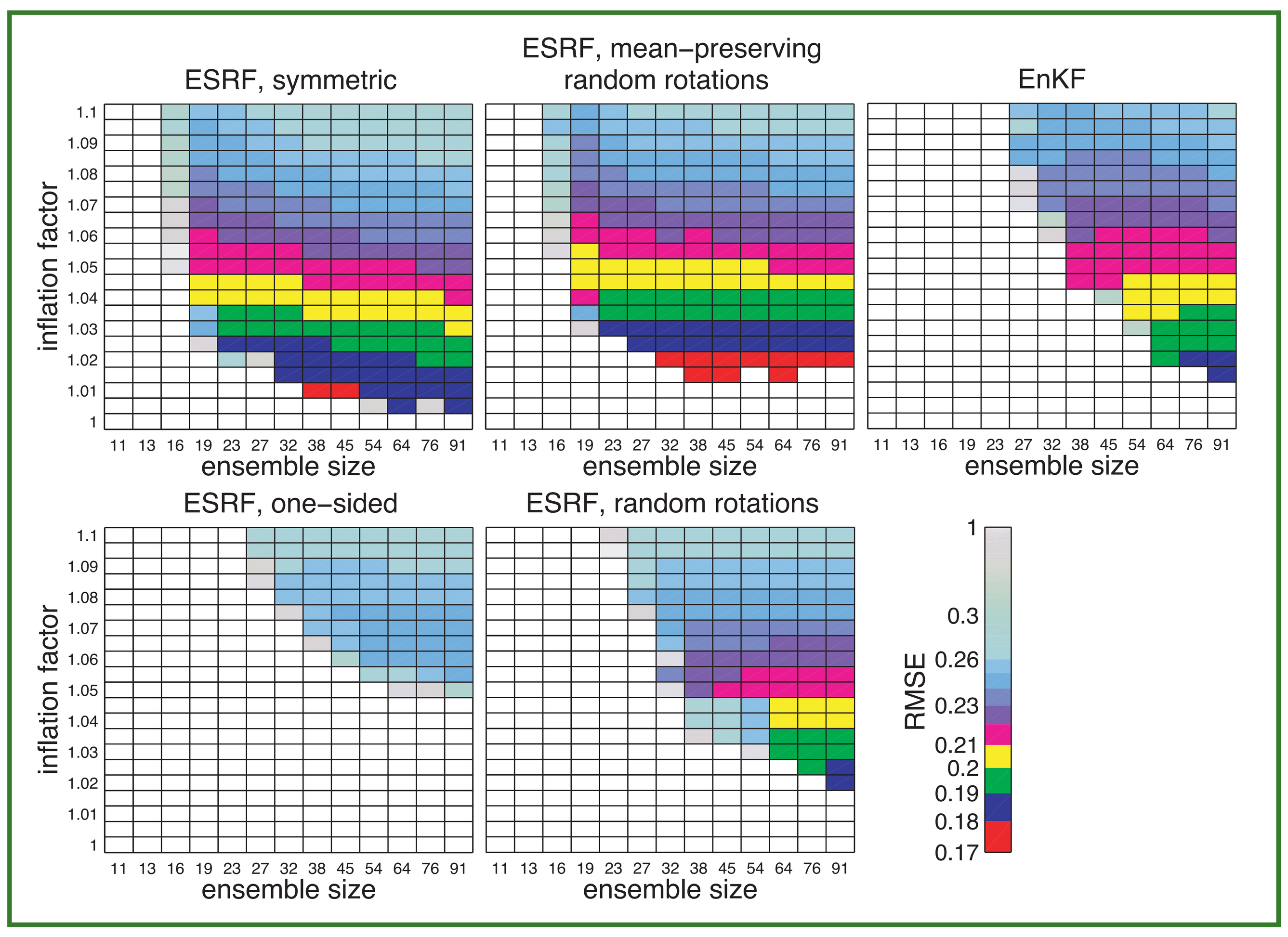
<<Figure 2

Normalised singular-value spectra of the ensemble anomalies for the LA model, averaged over 50 realisations:

(a) m = 30;

(b) m = 60.

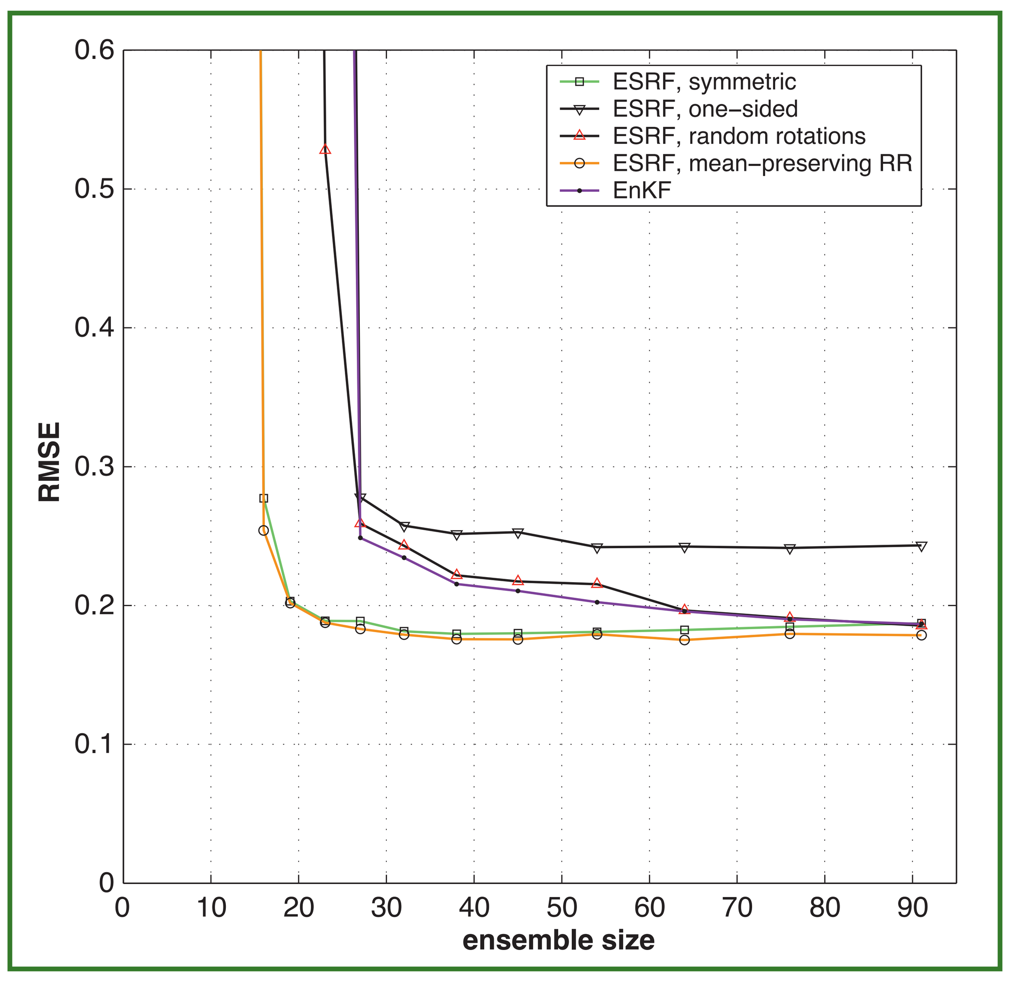
Figure 3



<<Figure 3

RMSE of different flavours of ESRF and EnKF for the L40 model averaged over a long model run.

Figure 4



<<Figure 4

The best RMSE from Figure 3 for a given ensemble size over all inflation factors.

References:

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