A deterministic ensemble Kalman filter

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We propose an ensemble Kalman filter (EnKF) without perturbed observations, referred to as the deterministic EnKF, or DEnKF. DEnKF is asymptotically equivalent to the ensemble square root filter (ESRF) in the case when the analysis correction is small; and readily permits the use of the traditional Schur product-based localisation schemes.

Theory

• Kalman filter:

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}(\mathbf{d} - \mathbf{H}\mathbf{x}^f),$$
$$\mathbf{P}^a = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}^f,$$
where $\mathbf{K} = \mathbf{P}^f\mathbf{H}^T(\mathbf{H}\mathbf{P}^f\mathbf{H}^T + \mathbf{R})^{-1}$

• Ensemble approach:

$$\mathbf{x} \longrightarrow \frac{1}{m}\sum_{i=1}^{m}\mathbf{X}_i \qquad \mathbf{P} \longrightarrow \frac{1}{m-1}\mathbf{A}\mathbf{A}^T \qquad (\mathbf{A} = \mathbf{X} - [\mathbf{x}, \ldots, \mathbf{x}])$$

• Traditional EnKF:

$$\mathbf{X}^a = \mathbf{X}^f + \mathbf{K}(\mathbf{D} - \mathbf{H}\mathbf{A}), \quad \mathbf{D}: \ \frac{1}{m-1}\mathbf{D}\mathbf{D}^T = \mathbf{R}$$

• ESRF:

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}(\mathbf{d} - \mathbf{H}\mathbf{x}^f),$$
$$\mathbf{A}^a = \mathbf{A}^f\mathbf{T}, \ \ \mathbf{T} = \left(\mathbf{I} - \frac{\mathbf{A}^{f\,T}\mathbf{H}^T\mathbf{M}^{-1}\mathbf{H}\mathbf{A}^f}{m-1}\right)^{1/2}, \quad \mathbf{M} \equiv \mathbf{H}\mathbf{P}^f\mathbf{H}^T + \mathbf{R},$$
or $\mathbf{A}^a = \mathbf{T}\mathbf{A}^f, \ \ \mathbf{T} = (\mathbf{I} - \mathbf{K}\mathbf{H})^{1/2} = \mathbf{I} - \frac{1}{2}\mathbf{K}\mathbf{H} - \frac{1}{8}(\mathbf{K}\mathbf{H})^2 + \ldots$

• Deterministic EnKF (DEnKF):

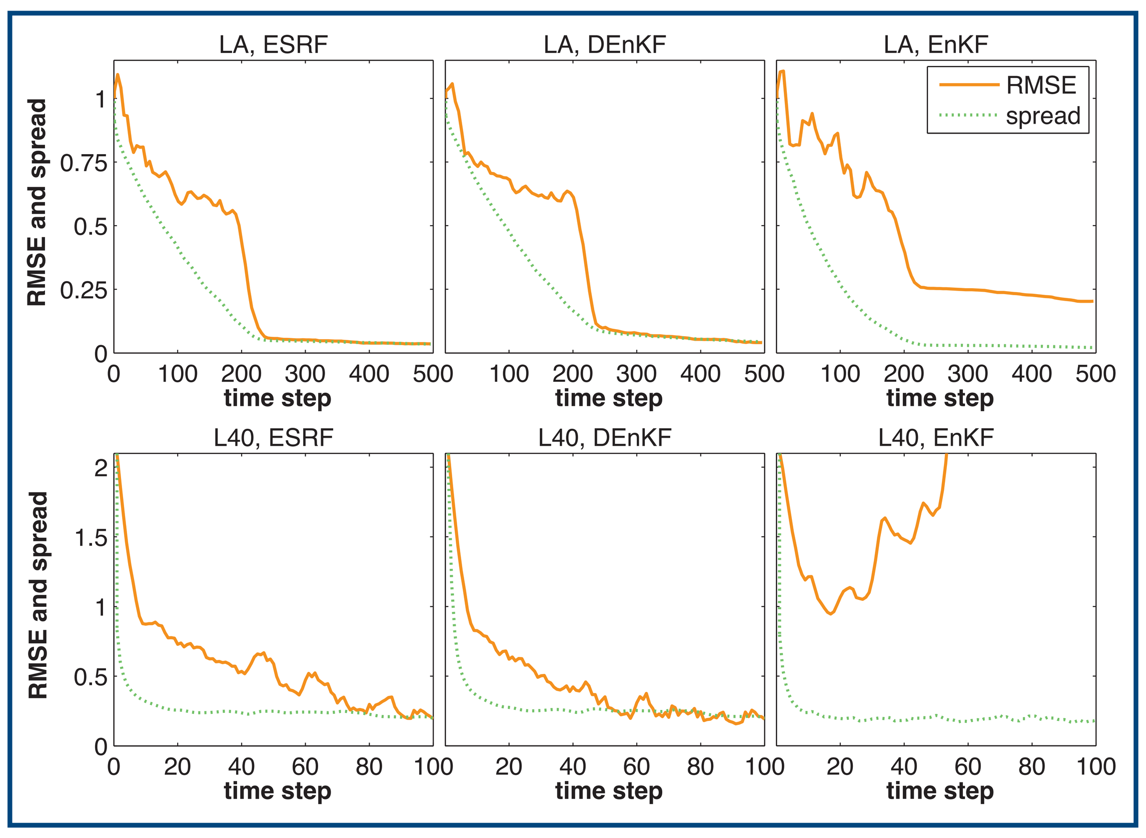
$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}(\mathbf{d} - \mathbf{H}\mathbf{x}^f),$$
$$\boxed{\mathbf{A}^a = \mathbf{A}^f - \frac{1}{2}\mathbf{K}\mathbf{H}\mathbf{A}^f.}$$

Numerical tests

Models:

- Linear advection (LA) model (Evensen, 2004), Figures 1 and 2.
- Lorenz-40 model (Lorenz and Emanuel, 1998; setup as in Whitaker and Hamill, 2002), Figures 3, 4 and 5.

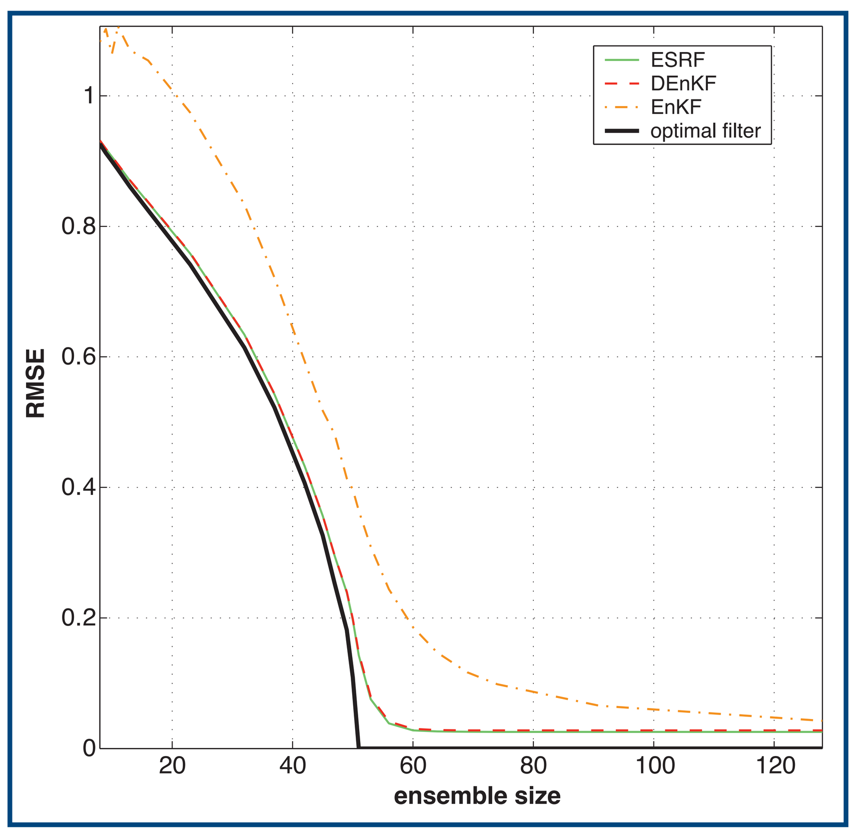
Figure 1



<< Figure 1

An example of RMSE and spread of ESRF, DEnKF and EnKF for LA and L40 models.

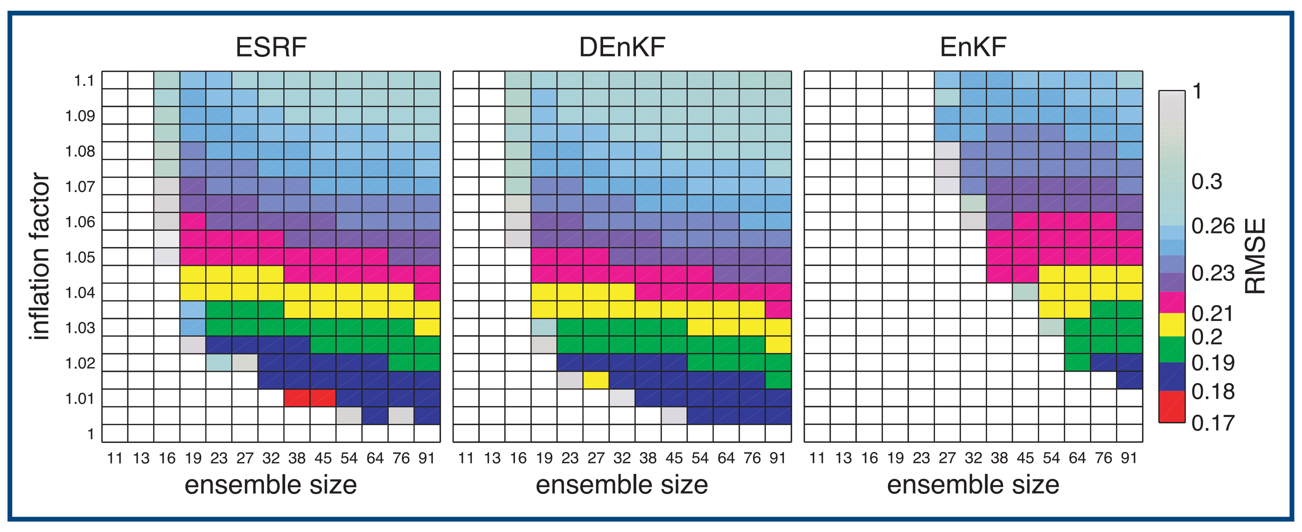
Figure 2



<< Figure 2

RMSE of ESRF, DEnKF and EnKF for the LA model, averaged for the time interval t = [900, 1000], and over 50 realisations.

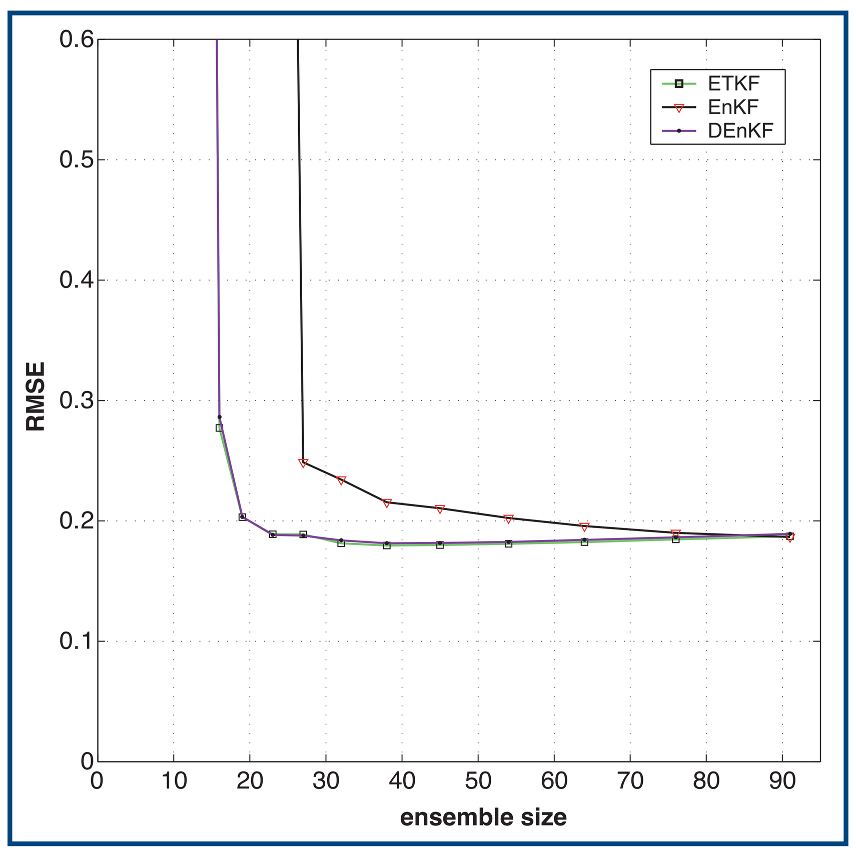
Figure 3



<< Figure 3

RMSE of ESRF, DEnKF and EnKF for the L40 model averaged over a long model run.

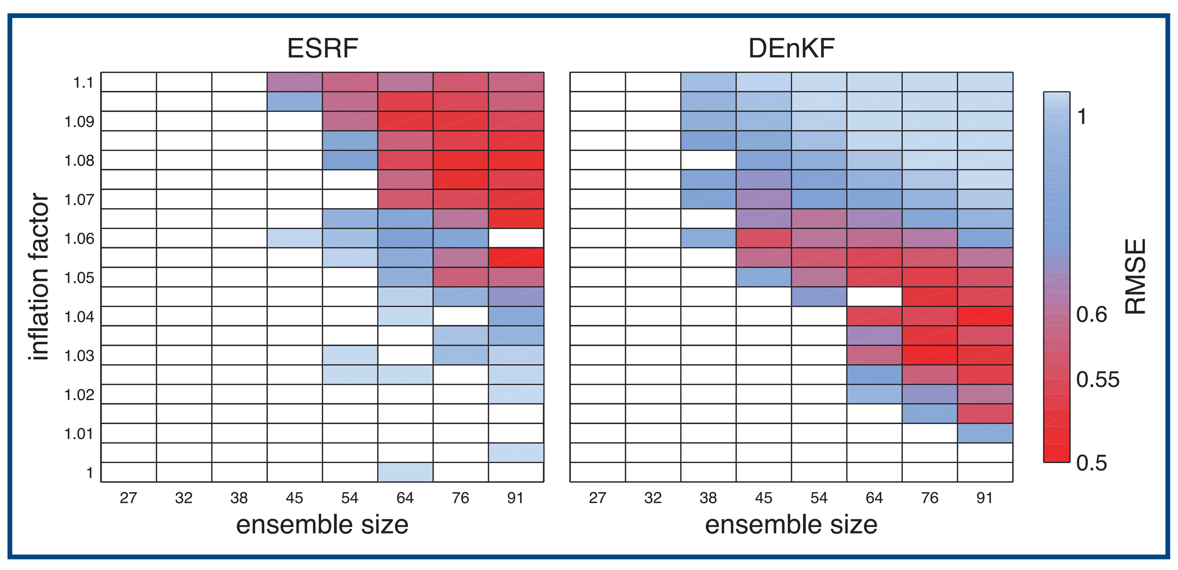
Figure 4



<< Figure 4

The best RMSE from Figure 3 for a given ensemble size over all inflation factors.

Figure 5



<< Figure 5

Comparison of convergence from the initial ensemble for the ESRF and DEnKF with L40 model in difficult conditions. Shows mean RMSE for time interval t = [200, 500], averaged over 50 realisations; 10 observations with observation error variance of 0.3 are assimilated every 2 time steps.

References:

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