

9. Summary of Results for Response Functions

9a Definitions and results

Impulse response functions can be a powerful way of describing the behaviour of the atmospheric CO₂ system (Oeschger and Heimann, 1983). Impulse response functions have been calculated as perturbations from two specific cases: an ‘equilibrium response’ in which the background was zero emissions and a ‘perturbation response’ in which the background was the emissions deduced from the S650 concentration profile. The responses were calculated by adding 10 GtC to the background emissions (in 1995 for the perturbation case), integrating forward and calculating the concentration differences from the background case.

Equilibrium response functions have been calculated using Models E, G, H, J, L and W. Perturbation calculations have been undertaken with Models E, H, J, L, R and W.

The response function, $G_a(t)$, can then be evaluated as

$$G_a(t) = 2.123[C(t) - C_{\text{background}}(t)]/10.0 \quad (9.1)$$

$G(t)$ is taken to be 1.0 at $t = 0$, defined as mid-1995 for the perturbation case. The most widely quoted ocean response function is

$$G_O(t) = 0.131 + 0.201e^{-t/362.9} + 0.321e^{-t/73.6} + 0.249e^{-t/17.3} + 0.098e^{-t/1.9} \quad (9.2)$$

from the ocean GCM of Maier-Reimer and Hasselmann (1987) relative to the pre-industrial state. This response function is used as the basis for describing oceanic uptake in Models T and W.

The equilibrium and ‘S650’ response functions are tabulated in Tables 9.1 and 9.2, and plotted in Figures 9.1 and 9.2 respectively.

Model	0	10	25	50	100	200	300	400	500
E	1.0	0.546	0.386	0.289	0.234	0.204	0.187	0.174	0.163
G	1.0	0.639	0.552	0.497	0.435	0.369	0.329	0.299	0.276
H	1.0	0.430	0.284	0.217	0.174	0.141	0.126	0.117	
J	1.0	0.560	0.410	0.316	0.248	0.204	0.185	0.172	0.163
L	1.0	0.643	0.459	0.354	0.302	0.254	0.228	0.212	0.202
W	1.0	0.629	0.442	0.309	0.225	0.176	0.155	0.143	0.135

Table 9.1. Equilibrium response function, at selected values of t in years, calculated relative to constant pre-industrial concentrations.

Model	0	10	25	50	100	200	300	400	500
E	1.0	0.624	0.493	0.419	0.401	0.425	0.408	0.389	
H	1.0	0.679	0.567	0.540	0.560	0.562	0.531		
J	1.0	0.628	0.507	0.435	0.401	0.393	0.376	0.359	0.348
L	1.0	0.695	0.555	0.495	0.477	0.459	0.424	0.399	0.386
R	1.0	0.786	0.616	0.531	0.488				
W	1.0	0.709	0.555	0.430	0.368	0.347	0.320	0.280	0.247

Table 9.2. Perturbation response function, calculated relative to S650.

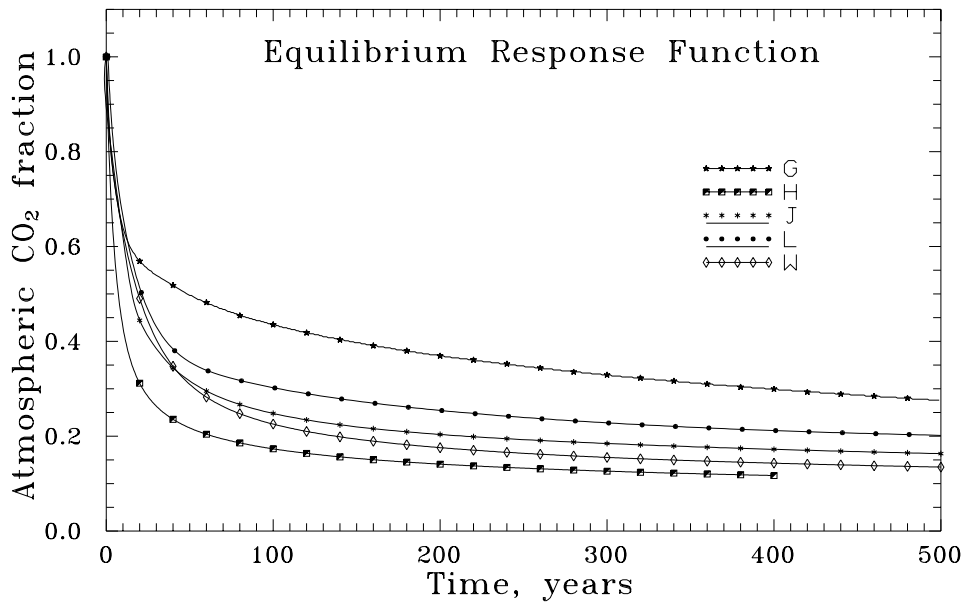


Figure 9.1. Impulse response relative to pre-industrial state.

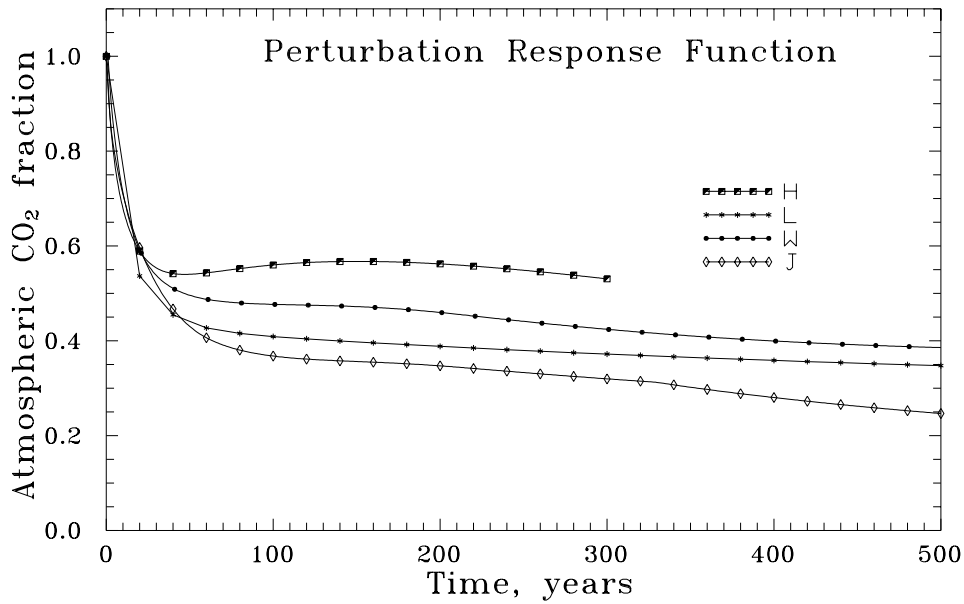


Figure 9.2. Impulse response relative to S650.

The calculation of global warming potentials requires the evaluation of the time-integrated CO_2 radiative forcing. With the radiative forcing, F_{CO_2} , of CO_2 (in Wm^{-2}) approximated by (Shine et al: IPCC, 1990, p41)

$$F_{\text{CO}_2} = 6.3 \ln \frac{C(t)}{C(0)}, \quad (9.3)$$

the integrated extra forcing per Gt of carbon is

$$\int_0^\tau \Delta F_{\text{CO}_2} dt' = \int_0^\tau \frac{6.3}{2.123 C(t')} G_a(t') dt' \quad (9.4)$$

Again $t' = 0$ is defined as mid-1995. Integrated forcings have been calculated for the five models listed above.

9b The reference response for GWP calculations

The Global Warming Potential (GWP) is a numerical index that is designed to assist in comparisons of the relative radiative importance of various greenhouse gases. The index has to be defined for specific time intervals of interest. The definition of the GWP of constituent X (for time horizon τ) is

$$\text{GWP}_{X,\tau} = \frac{\int_{t_X}^{\tau+t_X} \Delta F_X(t', t_X) dt'}{\int_{t_r}^{\tau+t_r} \Delta F_{\text{ref}}(t', t_r) dt'} \quad (9.5)$$

where the ΔF terms give the extra radiative forcing at time t' due to a unit input of constituent X at time t_X . Note that this definition has an implicit dependence on both t_X and t_r , the initial impulse times for X and the reference. These are important if non-linearities occur in either the emission-concentration or the concentration-radiative forcing relationship. The IPCC (1990) definition was to use CO_2 as the reference gas.

The difficulties with this definition were:

- Non-linearities in the response of CO_2 to emission changes mean that in equation (9.3) defining ΔF_{CO_2} for CO_2 , the term $G_a(t)$ (equation 9.1) depends on the background concentration profile.
- The concentration dependence of the radiative forcing of CO_2 (equation 9.3) will give an additional dependence on CO_2 profile for the CO_2 forcing.
- Uncertainties in the carbon budget translate into additional uncertainties in the CO_2 response (see Section 11). Specifically, the results depend on how the contemporary budget is balanced *vis-a-vis* the incorporation of a CO_2 -fertilisation effect.
- The three dependences above will propagate into *all* GWP calculations if the actual forcing from CO_2 is used as the reference case for GWP definitions.

In order to address these difficulties, the IPCC report on Radiative Forcing of Climate Change (IPCC, 1994, chapter 5) adopted the following procedure:

- The reference case (defining the denominator of 9.5) would be an idealised CO_2 radiative forcing.
- A particular background CO_2 concentration profile would be specified, so as to avoid the concentration dependence of both the response and the radiative effects.

- The background CO₂ concentration profile chosen as the reference was a constant concentration (at 354.17 ppmv) from 1990 onwards. (This avoids the potential dependence of t_r noted in connection with 9.5).
- The reference gas impulse response would be defined as the effect of a pulse release in 1995 as calculated with a representative model.
- On the basis of the results of the calculations presented in this report, Model J, the model described by Siegenthaler and Joos (1992), was chosen as the reference case.

The response function for the reference case is tabulated in Table 9.3 and plotted in Figure 9.3 (together with other response functions calculated using Model J).

Time	0	20	40	60	80	100	200	300	400	500
G_a	1.0	0.515	0.412	0.359	0.327	0.305	0.253	0.231	0.217	0.206

Table 9.3. Response function for Model J, calculated by adding a pulse in 1995 to the emissions required to give constant concentrations (354.17 ppmv) from 1990 onwards. Note that the differences from Table 9.1 arise from the higher background concentration which in turn requires non-equilibrium conditions at the time of the pulse.

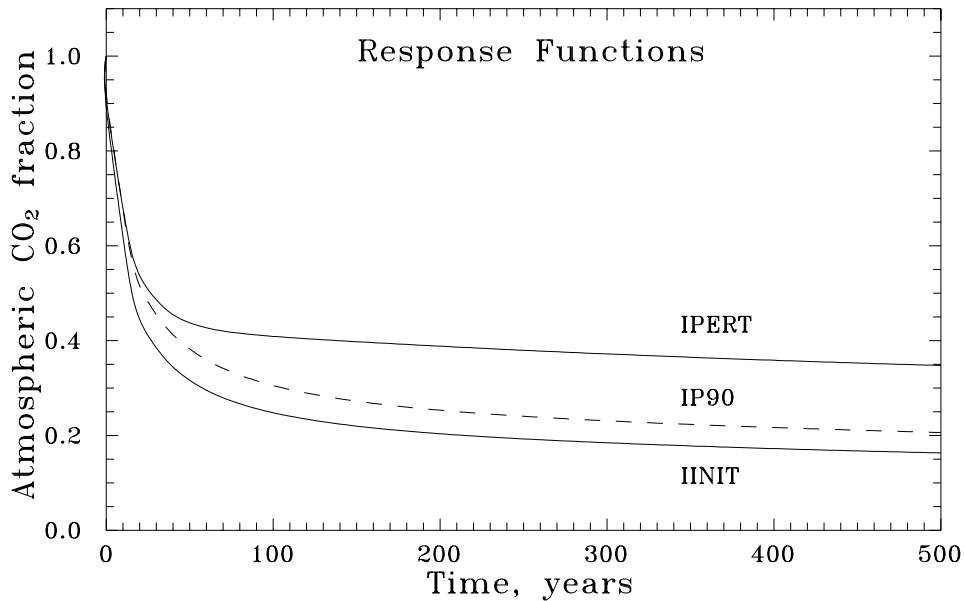


Figure 9.3. Response functions for Model J. IINIT is relative to the pre-industrial case, IPERT is relative to S650 and IP90 is the case defined as the reference for GWP, i.e., relative to constant concentrations from 1990.

The response functions for Model J were parameterised as sums of exponentials in the form

$$G_a(t) = a_0 + \sum_j a_j \exp(-t/\tau_j) \quad (9.6)$$

with the a_j and τ_j as specified in Table 9.4.

j	0	1	2	3	4
IINIT					
a_j	0.130164	0.333279	0.260540	0.165742	0.110275
τ_j	—	4.144656	18.587414	58.455562	414.152957
IPERT					
a_j	0.297144	0.202022	0.365643	0.135191	—
τ_j	—	3.054167	16.5172219	506.977719	—
IP90					
a_j	0.168312	0.261529	0.246715	0.197060	0.126483
τ_j	—	3.899498	18.664312	59.315997	415.962281

Table 9.4. Coefficients defining the parameterisation of the response functions calculated using Model J. IINIT is relative to the pre-industrial case, IPERT is relative to S650 and IP90 is the case defined as the reference for GWP, i.e., relative to constant concentrations from 1990.

9c. Combining response functions

The analysis in the previous sections has involved the response of the combined terrestrial-plus-oceanic system to perturbations in atmospheric CO₂ levels. However, it can be useful to study the separate responses of oceanic and terrestrial components, firstly as a basis for combining results from components of a single model and secondly for relating results of full models to the results of partial models. This section shows how sub-system responses are related to the total response.

The general equation governing concentration changes may be written as

$$\frac{d}{dt}N_a(t) = Q(t) - S_{\text{fert}}(t) - S_{\text{ocean}}(t) = Q(t) - \frac{d}{dt}N_b(t) - \frac{d}{dt}N_o(t) \quad (9.7)$$

where N_b denotes changes in terrestrial carbon without any land-use change.

Assume that we have a linear system consisting of atmosphere, ocean and biosphere. From the definition of the atmospheric impulse response function, $G_a(\tau)$, as a Green's function, we may write:

$$N_a(t) = \int_0^\infty G_a(\tau) Q(t - \tau) d\tau \quad (9.8a)$$

where $N_a(t)$ represents the excess atmospheric CO₂ mass at time t and $Q(t)$ the atmospheric CO₂ source.

Treating the terrestrial and oceanic components separately, the atmospheric concentrations are also linear in the sources — the sources for each component being the anthropogenic source

plus the carbon flux from the other component. We can define partial Green's functions, $G_{a:bio}$ and $G_{a:oc}$ such that:

$$N_a(t) = \int_0^\infty G_{a:bio}(\tau) [Q(t - \tau) - S_{ocean}(t - \tau)] d\tau \quad (9.8b)$$

and

$$N_a(t) = \int_0^\infty G_{a:oc}(\tau) [Q(t - \tau) - S_{fert}(t - \tau)] d\tau \quad (9.8c)$$

where $G_{a:oc}$ and $G_{a:bio}$ are the respective responses of the oceanic and terrestrial components to a pulse source.

The equations (9.7, 9.8a,b,c) involving convolution integrals can be solved by taking Laplace transforms and solving the resulting algebraic equations for G_a . In practice, the partial impulse response functions are given in numerical form, calculated using the models for each component separately and applying pulse sources. By numerical fitting these can be represented as a series of exponential functions (plus possibly a constant offset). The series can be easily transformed and algebraically solved for the Laplace transform of G_a . Since only exponentials are involved, rational functions of p result. These are readily transformed back into a series of exponentials.

Using h , or more specifically $h(p)$, to denote the Laplace transform of a function $H(t)$ we obtain the transforms of equations (9.8a–c) as equations (9.9a–c)

$$n_a = g_a(p) q(p) \quad (9.9a)$$

$$n_a = g_{a:bio}(q - pn_o) \quad (9.9b)$$

$$n_a = g_{a:oc}(q - pn_o) \quad (9.9c)$$

and the transform of (9.7) is

$$pn_a = q - pn_b - pn_o \quad (9.10)$$

Eliminating n_o and n_b gives:

$$pn_a = n_a/g_{a:bio} + n_a/g_{a:oc} - q(p) \quad (9.11)$$

so that for the full atmospheric response function, $G_a(t)$, we have from (9.10)

$$g_a = p^{-1} \left[\frac{1}{p g_{a:oc}} + \frac{1}{p g_{a:bio}} - 1 \right]^{-1} = \frac{g_{a:oc} g_{a:bio}}{g_{a:oc} + g_{a:bio} - p g_{a:oc} g_{a:bio}} \quad (9.12)$$

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