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TECHNIQUES FOR DETERMINING SURFACE SOURCES FROM  
SURFACE OBSERVATIONS OF ATMOSPHERIC CONSTITUENTS

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Abstract

This report considers techniques for using an atmospheric transport model to deduce surface source strengths given surface concentrations as a function of latitude. In particular it is shown that the class of 'source-determination' problems including the problem of determining seasonal CO<sub>2</sub> sources is well-defined and that formally a unique source solution can be expected. Specific computational techniques are described and a number of numerical examples related to the seasonal variation of CO<sub>2</sub> are presented.

## 1. Introduction

This report describes a number of mathematical and computational techniques for determining the strengths of the sources and sinks of atmospheric constituents, assuming that these sources are at the surface of the earth and that the concentration of the constituent is also known at the earth's surface. The most important example of this type of calculation is the deduction of seasonally varying source strengths of  $\text{CO}_2$  from the observed annual cycle of  $\text{CO}_2$  at the earth's surface. The earliest attempt to perform this calculation was by Bolin and Keeling (1963). This calculation was limited in accuracy, firstly because of the limited data sets available at that time and secondly because they tried to determine simultaneously both source strengths and atmospheric transport coefficients. Their calculation was based on a spectral representation of a one-dimensional (latitude only) model of atmospheric transport.

A more extensive calculation of surface  $\text{CO}_2$  sources was undertaken by Pearman and Hyson (1980). They used a two-dimensional (i.e. zonally averaged) model of atmospheric transport (Hyson et al., 1980). The main transport process was advection and observed wind-field data were used in the model. There was an additional diffusive transport that was tuned by using observations of the distribution of  $\text{CCl}_3\text{F}$ . An alternative approach to the question of the seasonally varying  $\text{CO}_2$  sources has recently been described in the work of Fung et al. (1983). They employed data on Net Primary Production to estimate the biospheric sources and used these sources in their three-dimensional model calculations. These calculations produced large differences in seasonal amplitudes between oceanic and continental regions. The amplitudes in oceanic regions were in agreement with observations at these sites as were those of Pearman and Hyson (1980) (by construction) but because of the land-sea differences, Fung et al. obtained this agreement using sources that were roughly twice the strength of those used by Pearman and Hyson (1980). This discrepancy has been discussed by Enting and Pearman (1984).

The layout of the remainder of this report is as follows:

Section 2 describes the mathematical basis of the source determination problem in terms of Green's functions for the differential equations obtained by expressing the transport model in a spatial finite difference formalism. This gives an explicit solution that demonstrates the existence of a unique source function in all but exceptional singular cases. A mathematical expression of the separation of the atmospheric and ocean modelling aspects of the  $\text{CO}_2$  problem is also described.

Section 3 describes numerical techniques for determining the surface sources, given an atmospheric transport model and a specification of the surface concentrations as a function of time. The main requirements for the applicability of the theory from Section 2 and the numerical techniques in Section 3 are that the concentrations be specified (as functions of time) at those grid points at which the sources are unknown and that the remaining contributions to the atmospheric model should be directly proportional to the concentrations. These requirements are satisfied for the  $\text{CO}_2$  problem if the surface concentrations are specified because all the sources are at the surface, and the only other processes

are the transports which are, for all passive constituents, modelled as being proportional to the concentrations. The requirements would also be satisfied for a constituent such as methane if the destruction coefficients in the bulk of the atmosphere were assumed to be known. This chemical destruction is generally modelled as being proportional to the methane concentration so that given the rate coefficients, a specification of surface concentrations of methane would allow the deduction of the net source strength at the earth's surface.

Section 4 gives a brief summary of the revised two-dimensional transport model developed within the Division of Atmospheric Research of CSIRO, Australia (Enting, 1984) and Section 5 shows some trial solutions that were obtained for the CO<sub>2</sub> system using this model, illustrating some of the numerical characteristics of the computational techniques, including discussion of the error analysis. The final section summarises the results that have been presented and further reviews the range of problems to which the techniques may be applied. Appendix A gives additional details of a special case of the Green's function formalism and Appendix B lists the spectral form of the transport coefficients that was used in the numerical examples.

## 2. Mathematical Formalism

The general solution of the 'source deduction' problem is most readily expressed in terms of Green's functions defined in terms of the set of ordinary differential equations that are used to model the atmospheric transports in terms of transport parameters (A), concentrations (C) and sources (S).

The general expression that is considered is:

$$\frac{dC_i}{dt} - \sum_j A_{ij}(t) C_j(t) = S_i(t) \quad (2.1a)$$

or

$$\sum_j L_{ij} C_j = S_i \quad (2.1b)$$

where the set of equations arises from either a finite difference representation of the spatial derivatives or a truncation of the spectral representation of the spatial behaviour. The indices  $i, j$  range over all spatial grid points (or over all spatial spectral components if a spectral representation is used).

The terms involved in the  $A_{ij}(t)$  will generally represent the transports but they will also include any sinks where the loss rate is proportional to the concentration, (e.g. radioactive decay or first order chemical processes). Under the two conditions given below, the differential operator  $L$  has an inverse  $G$  called the Green's function operator such that

$$C_i = \sum_j G_{ij} S_j \quad (2.2)$$

As might be expected, the inverse of the differential operator  $L$  is an integral operator. Equation 2.2 is a symbolic representation of

$$C_i(t) = \sum_j \int_a^b G_{ij}(t, t') S_j(t') dt' \quad (2.3)$$

where  $a$  and  $b$  are the values at which the conditions (2.4) apply.

The sufficient conditions for the existence of the Green's function are (Coddington and Levinson, 1955)

- (i) the conditions on the initial and final values can be expressed in terms of two matrices  $M$  and  $N$  as:

$$\sum_j [M_{ij} C_j(a) + N_{ij} C_j(b)] = 0, \text{ for all } i \quad (2.4)$$

and

- (ii) if  $S_i(t) \equiv 0$  then the differential equations have no non-trivial solutions that satisfy the conditions (2.4).

The two most important types of conditions are (a) zero initial conditions, i.e.  $C_i(a) = 0$  for all  $i$ , and (b) periodicity conditions  $C_i(a) = C_i(b)$  for all  $i$ .

For constituents that are conserved in the atmosphere, condition (ii) above is violated if periodic boundary conditions are involved because the constant concentration solution is possible when  $S_i \equiv 0$ . This case must be treated using some reduced set of equations - an outline is given in Appendix A.

The 'source-deduction' problem involves dividing the grid points into two sets,  $\alpha$  and  $\beta$ , where for  $j \in \alpha$  the sources (i.e. the  $S_j$ ) are unknown but the concentrations are known while for  $j \in \beta$  the sources are known but the concentrations are unknown.

Expanding equation 2.1 gives

$$\sum_{j \in \alpha} L_{ij} C_j + \sum_{p \in \beta} L_{ip} C_p = S_i, \quad i \in \alpha \quad (2.5a)$$

$$\sum_{j \in \alpha} L_{qj} C_j + \sum_{p \in \beta} L_{qp} C_p = S_q, \quad q \in \beta. \quad (2.5b)$$

The transport terms within the set  $j \in \beta$  are described by the term in (2.5b) involving  $L_{qp}$  so that the known source  $S_q$  and the transports of known  $C_j$  from the set  $\alpha$  act as an 'effective source' for set  $\beta$ , i.e.

$$\sum_{p \in \beta} L_{q,p} C_p = S_q - \sum_{j \in \alpha} L_{qj} C_j, \quad q \in \beta. \quad (2.6)$$

This set of equations describing transports within the set  $\beta$  will have a Green's function operator  $G_{rp}^*$  such that

$$C_r = \sum_{p, q \in \beta} G_{rp}^* L_{q,p} C_p = \sum_{q \in \beta} G_{rp}^* \left( S_q - \sum_{j \in \alpha} L_{qj} C_j \right), \quad r \in \beta. \quad (2.7)$$

Even in the case of conserved tracers it is not necessary to use a reduced set  $\beta'$  because the operator  $L$  acting on the set  $\beta$  is not conservative. In particular if the terms on the right of (2.6) are zero, i.e. no sources in set  $\beta$  and zero concentrations in set  $\alpha$ , there will be a loss of a tracer from region  $\beta$  into region  $\alpha$ .

Substituting the result (2.7) back into (2.5a) gives

$$S_i = \sum_{j \in \alpha} \left[ L_{ij} - \sum_{q, r \in \beta} L_{ir} G_{rq}^* L_{qj} \right] C_j + \sum_{r, q \in \beta} L_{ir} G_{rq}^* S_q. \quad (2.8)$$

The main use of equation (2.8) is as a demonstration of the existence of a unique solution to the 'source-deduction' problem.

As indicated in the introduction one of the main problems to which this formalism can be applied is the determination of the surface sources of  $CO_2$  given observations of surface concentrations. In this case known 'fixed' sources above the surface are zero, i.e.  $S_q = 0, q \in \beta$  so that the last sum in (2.8) vanishes. The result 2.8 shows that the total net surface source of  $CO_2$  is determined as a functional of the surface concentration functions. Thus total surface source is made up of a biospheric contribution and an ocean contribution. Since however the ocean contribution is generally modelled by having the uptake determined as a response to the  $CO_2$  concentration at the surface, the net ocean uptake rate will also be a functional of the surface concentrations. The net biospheric source can be obtained by taking the difference between the total source and the ocean source. There are two terms in the result that are of further interest:

- (a) The zero frequency term: If the periodic case is being investigated it is necessary to remove the mean atmospheric  $CO_2$  increase from the data sets. It is not, however, necessary to remove the time-averaged interhemispheric gradient and from this gradient the formalism described above will produce a net interhemispheric source imbalance giving the extent to which the mean annual source at a point differs from the global mean source. (As will be seen from the results in Section 5 the 'zero-frequency' relation connecting interhemispheric gradients to interhemispheric transport is not entirely independent of the seasonal variations because of covariance effects between the seasonal cycle and the transports.)
- (b) The 1 year period term: This component is generally dominant and it can be useful to examine it in detail, particularly in the context of error analysis. The analysis can be presented in a simple graphical form using the 'phasors' that are commonly used in electrical engineering. Any sinusoidal variation



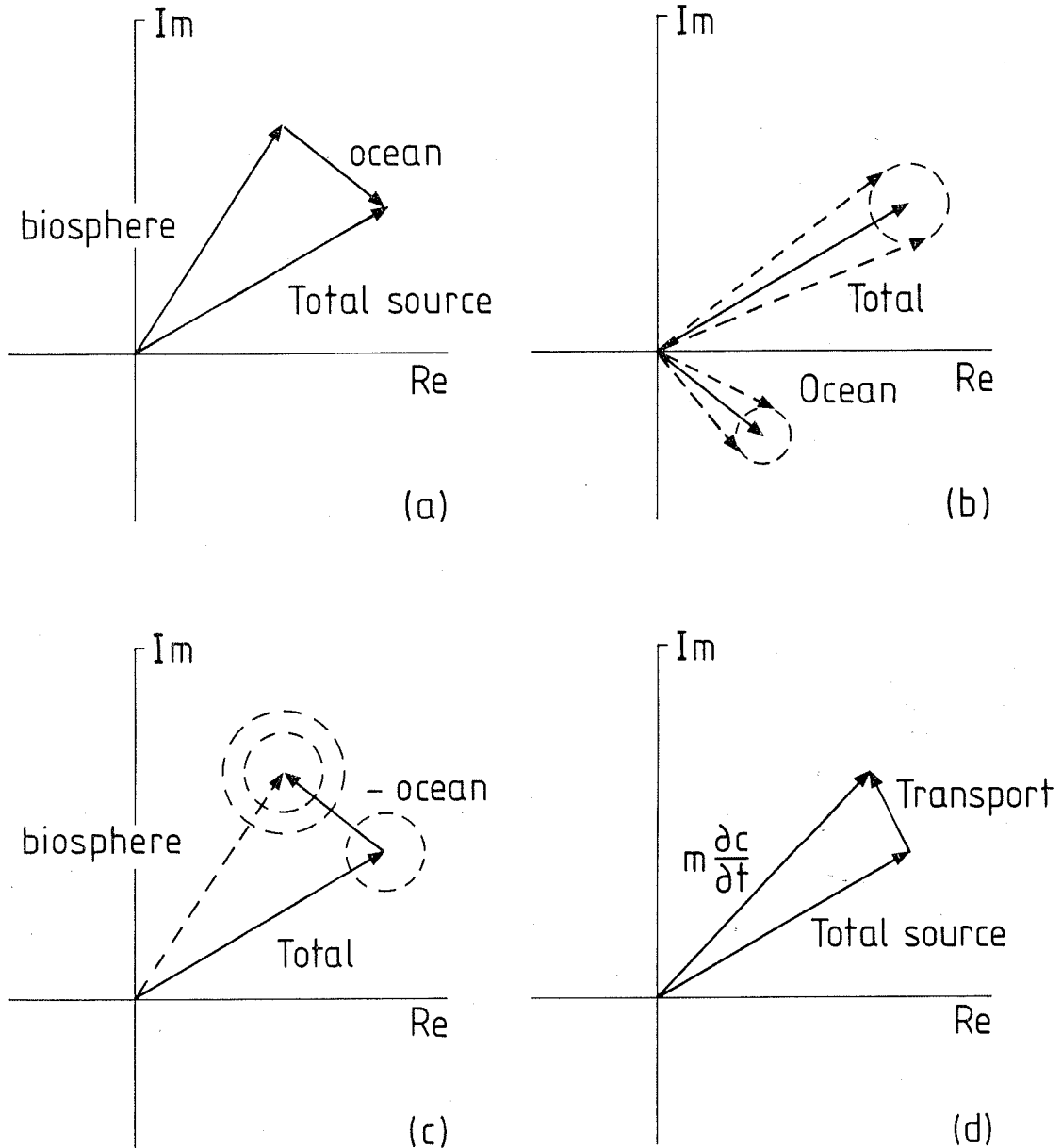


Figure 2.1: Phasor diagrams for amplitude and phase relations between the phases of sources. The lengths of the vectors represent the amplitudes and the angles with respect to the horizontal represent the phases (relative to some arbitrary zero). (a) The total  $\text{CO}_2$  source is the sum of the oceanic and biospheric sources; (b) The total source determined by the techniques of section 3 will be subject to various uncertainties due to the uptake model and the surface concentration data; (c) These uncertainties combine when the biospheric source is deduced by subtracting the ocean source from the total source; (d) Similar phasor relations describe a local rate of change of  $\text{CO}_2$  concentrations as a sum of transport and local source.

$\text{Acos}(2\pi t + \phi)$  is regarded as the real part of a complex quantity  $\text{Aexp}(2\pi it + i\phi)$ , factored as  $\text{Aexp}(i\phi) \cdot \exp(2\pi it)$ . Sums of sinusoidal variations can be represented by the sums of the exponentials since the real part of the sum is the sum of the real parts. The phase relations between the components can be analysed by neglecting the  $\exp(2\pi it)$  factor which is common to all terms and using the conventional graphical vector representation of complex numbers to indicate the  $\text{Aexp}(i\phi)$  factors. Figure 2.1 shows a set of hypothetical phasor representations of the relation between the sources. Figure 2.1a shows the vectors for biospheric source, ocean source and total source, illustrating how the total source is the vector sum of the other two sources. In actual calculations, each quantity will have a range of uncertainty - Figure 2.1b indicates the uncertainties in the total source and the ocean source, these being the two quantities that are calculated directly from the surface concentrations. Figure 2.1c indicates schematically how these uncertainties combine when calculating the biospheric source as a difference. The exact situation is somewhat more complicated than shown in these diagrams. Some of the uncertainties shown in Figure 2.1b will be due to the models - the atmospheric transport model and the ocean uptake model for total and oceanic sources respectively. When calculating the biospheric source these uncertainties can be regarded as independent. However in each case there will be uncertainties in the calculated sources due to uncertainties in the observed values of the surface concentrations. These uncertainties are not independent and the assessment of the final uncertainty must be obtained as a single calculation involving both atmospheric and ocean modelling.

As well as the phasor representation of the sources it is possible to obtain similar representations for mass balance in particular regions. Taking a factor  $m_i$  representing the number of moles of air associated with grid point  $i$ , equation 2.1a transforms to

$$\frac{d}{dt} m_i C_i = \sum_j m_i A_{ij}(t) C_j(t) + m_i S_i \quad (2.9)$$

where each term is in moles per year if  $C_i$  is a mixing ratio. Figure 2.1d shows a phasor representation of a single frequency component of equation (2.9) (again the 1 year period is of most interest) showing how local changes are a combination of transports and local sources.

### 3. Computational Techniques

The Green's function formalism given in the previous section is mainly useful for demonstrating the existence of unique solutions to the source deduction problem. Since the Green's function techniques involve integral operators they would not be particularly easy to use, even if there were a convenient way to construct the operators. In practice there is a simple direct technique based on numerical integration that is found to converge rapidly to the solutions for the sources.

The procedure is based on the same division of the grid points into two sets  $\alpha$  and  $\beta$  as in the previous section. For set  $\alpha$  the sources are unknown but the concentrations are known while for set  $\beta$  the concentrations are unknown but the sources are known - in many problems (e.g.  $\text{CO}_2$ ) these known sources are zero. The procedure is then to integrate the differential equations for set  $\beta$  numerically. If the system is one that has zero initial concentrations then the procedure for deriving sources can begin immediately. In the case of periodic boundary conditions the equations must be integrated through a number of cycles (typically 1 or 2) until a periodic solution is approached sufficiently closely. The equations for set  $\beta$  can be integrated numerically since the only terms appearing in these differential equations are:

- (i) the  $S_j$ ,  $j \in \beta$  which are assumed known;
- (ii) the  $C_j$ ,  $j \in \alpha$  which are assumed known;
- (iii) the  $C_j$ ,  $j \in \beta$  which are found by integrating the equations numerically so that the  $C_j$ ,  $j \in \beta$  are found for a succession of time steps.

Once the appropriate solutions for  $C_j$ ,  $j \in \beta$  have been found then the solutions for the sources can be found from

$$S_i = \frac{\partial C_i}{\partial t} - \sum_j A_{ij} C_j, \quad i \in \alpha \quad (3.1)$$

which involves only

- (i)  $C_i$ ,  $\frac{\partial C_i}{\partial t}$ ,  $i \in \alpha$ ;  $C_i$  is assumed known (and differentiable)
- (ii)  $C_j$ ,  $j \in \beta$ , obtained by numerical integration.

As a simple example of the way in which this can be implemented, the following algorithm description (based closely on PASCAL code) can be used. It assumes 4 procedures:

- (i) model(C,diffC,t) which has input C, the vector of concentrations and output diffC, the vector  $\sum A_{ij}(t)C_j$ ;
- (ii) eval(i,t) which gives  $C_i(t)$  for  $i \in \alpha$  at time t, i.e. the known concentration;
- (iii) cderiv(i,t) which gives  $\frac{\partial C_i}{\partial t}$  for  $i \in \alpha$  at time t;
- (iv) output(s,t) which takes a set of instantaneous sources strengths  $S_i(t)$ ,  $i \in \alpha$  and prints them (or stores them for Fourier analysis etc.).

The code implements a predictor corrector formalism for doing the time integration.

```

t : = t0 ;
while t < t max do
  begin
    model(C, Cdf1, t) ;
    for i ∈ β do
      C[i] : = C[i] + tstep*Cdf1[i] ;
    for i ∈ α do
      begin
        S[i] = cderiv(i,t) - Cdf1[i] ;
        C[i] = cval(i,t + tstep) ;
      end ;
    output(S,t) ;
    t : = t + tstep ;
    {calculate corrector step}
    model (C, Cdf2, t) ;
    for i ∈ β do
      C[i] : = C[i] + 0.5* (Cdf2[i] - Cdf1[i])*tstep ;
    end .

```

In this example, once the functions `cval` and `cderiv` and the output are defined only 6 additional lines (marked with \*) of code are needed to convert a predictor-corrector integration procedure into a source deduction procedure. Similar changes would be required in other integration procedures. For periodic systems such as the seasonal CO<sub>2</sub> cycles, the alternative to time stepping integration is to use a spectral representation of the time dependence. If the various functions are expanded as

$$S_i(t) = \sum_n S_{jn} \exp(2\pi nit) \quad (3.2a)$$

$$C_j(t) = \sum_n C_{jn} \exp(2\pi nit) \quad (3.2b)$$

$$A_{ij}(t) = \sum_n A_{ijn} \exp(2\pi nit) \quad (3.2c)$$

then the set of differential equations obtained from the spatial finite difference scheme becomes a set of algebraic equations:

$$2\pi in C_{jn} = \sum_m \sum_k A_{jkm} C_{k,n-m} + S_{jn} \quad (3.3)$$

where  $C_{jn}$  is known (and denoted  $q_{j,n}$ ) for  $j \in \alpha$  and  $S_{jn}$  is known for  $j \in \beta$ . For the case of centred difference schemes, this set of equations is likely to be diagonally dominant and the Gauss-Jacobi iterative technique is a possible way of obtaining solutions without having to explicitly invert the transport operator for the non-surface sites.

Equation (3.3) is written as

$$2\pi i n C_{j,n} - A_{jjo} C_{jn} = \sum_m \sum_k B_{jkm} C_{k,n-m} + S_{jn} \quad (3.4)$$

where

$$B_{jkm} = 0 \quad \text{if } j = k \text{ and } m = 0 \quad (3.5a)$$

$$= A_{jkm} \quad \text{otherwise.} \quad (3.5b)$$

This is used as the basis of an iterative solution

$$C_{jn}^{(p+1)} = (2\pi i n - A_{jjo})^{-1} \left( \sum_{km} B_{jkm} C_{k,n-m}^{(p)} + S_{jn} \right) \quad (3.6)$$

for  $j \in \beta$

$$\text{where } C_{k,m}^{(p)} = q_{k,m} \text{ for all } p \text{ if } k \in \alpha \quad (3.7a)$$

$$\text{and } C_{k,m}^{(0)} = 0 \quad \text{if } k \in \beta \quad (3.7b)$$

For  $j \in \alpha$  the sources are determined by

$$S_{jn} = 2\pi i n q_{jn} - \sum_m \sum_k A_{jkm} C_{k,n-m} \quad j \in \alpha \quad (3.8)$$

In order to implement the computational scheme based on equations (3.6) and (3.8) a sufficient computational capacity is the ability to compute  $\sum_k A_{jkm} \zeta_k$  as a field of  $\eta_j$  for any specified  $m$ .

The value  $A_{jjo}$  can be extracted from the sum  $\sum_k A_{jkm} \zeta_k$  by setting  $m = 0$ ,  $\zeta_j = 1$ ,  $\zeta_k = 0$  for  $k \neq j$  and evaluating the sum. Since each  $A_{jjo}$  need only be computed once for the whole calculation rather than once per iteration, the inefficiency involved in this procedure is not particularly serious and it has the considerable advantage that it allows the 'source-deduction' procedure to be formulated in a manner independent of the finite difference scheme that is used. The sum in (3.6) is then evaluated as

$$\sum_m \sum_k B_{jkm} C_{k,n-m}^{(p)} = \sum_k \left( \sum A_{jk,m} C_{k,n-m}^{(p)} \right) - A_{jjo} C_{jn}^{(p)} \quad (3.9)$$

where the bracketed sum is the standard sum with the field  $\zeta_k$  corresponding to a particular frequency component of the concentration field.

It is possible to use a single computer routine to perform the essential part of the evaluation for both this standard sum and the sums needed in the time-stepping integration.

This is demonstrated by noting that the transport equation (described in Section 4) and all practical finite difference representations of this equation depend linearly on the transport coefficients which we denote as  $\phi_r$ .

$$\text{Thus } A_{jk}(t) = \sum_r T_{jkr} \phi_r(t) \quad (3.10)$$

$$\text{and if } \phi_r(t) = \sum_m \phi_{rm} \exp(2\pi i m t) \quad (3.11)$$

$$\text{then } A_{jkm} = \sum_r T_{jkr} \phi_{rm} \quad (3.12)$$

A routine that evaluates  $\sum_k \sum_r T_{jkr} \zeta_k \gamma_r$  for arbitrary sets of fields  $\zeta_k$  defined on all grid points and for arbitrary sets of transport coefficients  $\gamma_r$  can be used in both the spectral form of the source deduction problem and the time stepping formalism. To illustrate this the term corresponding to  $\sum_{kr} T_{jkr} \zeta_k \gamma_r$  is enclosed in brackets { } in the expressions below.

- (i) The spectral form of source deduction requires

$$\sum_k A_{jkm} \zeta_k = \left\{ \sum_r T_{jkr} \zeta_k \phi_{rm} \right\}, \text{ explicitly.} \quad (3.13)$$

- (ii) the time stepping forms require

$$\begin{aligned} & \sum_k A_{jk}(t) C_k(t) \\ &= \left\{ \sum_k \sum_r T_{jkr} C_k(t) \phi_r(t) \right\} \end{aligned} \quad (3.14a)$$

$$= \sum_m \exp(2\pi i m t) \left\{ \sum_k \sum_r T_{jkr} C_k(t) \phi_{rm} \right\}. \quad (3.14b)$$

The two forms of the sum for the time stepping are of comparable computational complexity. If there are  $N$  grid points then there are approximately  $4N$  transport coefficients and if  $M$  spectral components are used then evaluating the  $\phi_r(t)$  requires  $4NM$  multiplications. The sum in brackets requires  $\sim 6N$  multiplications if a centred difference scheme is used so the first expression requires  $4NM + 6N$  multiplications and the second requires  $6NM + MN$ . The first form is faster (at the expense of having to explicitly store the  $N \phi_j(t)$  values). In many calculations it may be sufficiently accurate to update the  $\phi_j(t)$  in a

stepwise fashion rather than at every time step and this will of course accelerate the computations considerably. Explicitly evaluating the Fourier series first (as in 3.14a) rather than as the last step (as in 3.14b) is essential when the spatial variation is represented in a spectral form as described in Section 4. In this case, truncation of the spatial sum can lead to spurious small negative values for  $K_{pp}$  and  $K_{yy}$ . This can only be detected when the final  $\phi(t)$  is evaluated. If negative  $K_{pp}$ ,  $K_{yy}$  values are detected they must be replaced by small positive values as otherwise the transport equation would become unstable.

#### 4. The Transport Model

The revised Division of Atmospheric Research two-dimensional atmospheric transport model has been described in detail by Enting (1984). This section simply summarises the main features of the model and describes its use in the source-determination calculations.

- (i) The transport model is based on a finite difference representation of the two dimensional transport equation:

$$\begin{aligned} m \frac{\partial C}{\partial t} = & \frac{\partial}{\partial p} \left[ m K_{pp} \frac{\partial C}{\partial p} + (m K_{py} - \psi) \frac{\partial C}{\partial y} \right] \\ & + \frac{\partial}{\partial y} \left[ m K_{yy} \frac{\partial C}{\partial y} + (m K_{py} + \psi) \frac{\partial C}{\partial p} \right] \end{aligned} \quad (4.1)$$

'Reduced' or dimensionless units are used so that the vertical coordinate  $p$  is a reduced pressure,  $0 \leq p \leq 1$ ,  $y = \sin(\text{latitude})$  and  $\psi$ , the atmospheric mass stream function is in atmospheric masses per year. The mean density in  $p$ - $y$  coordinates is  $m = \frac{1}{2}$ . The diffusion coefficients are related to those in other coordinate systems by appropriate tensor transformations (see Enting, 1984). The concentrations  $C$  are taken as being some fixed multiple of the mixing ratios.

- (ii) The finite difference representation uses an equally spaced grid in the  $p$  and  $y$  directions. The computer implementation is written so that all subprograms are independent of the number of grid points. The actual resolution is specified in a single common block (DEFN) and all the two-dimensional fields are stored in common blocks. To enlarge the amount of storage it is (in many Fortran systems including that of DAR) sufficient to change the sizes of all common blocks within one BLOCK DATA subprogram.
- (iii) The spatial differencing scheme uses centred differences and is (apart from change of coordinates) equivalent to that given by Miller et al. (1981), reexpressed to improve the computational efficiency. The standard time integration technique is the predictor-corrector technique as illustrated in the schematic code in Section 3.

- (iv) The model is implemented as a library of subroutines so as to modularise the computations. The most important routine, TRANS, performs the summation  $\sum \sum T_{jkr} \zeta_k \phi_r$  described in Section 3. This is fundamental to normal time stepping integration, the time stepping form of the source deduction problem and the spectral form of the source deduction problem. The full time stepping sum of transport terms defined by equation (3.14a) together with any known source terms is implemented by routine MODEL which combines a call of TRANS and a call of a routine SOURCE. The library provides alternative versions of SOURCE for different tracers. Similarly the source deduction routine DEDUCE (which is essentially a Fortran version of the code shown in Section 3) takes the names of routines Cval, Cderiv and output as external subprogram names so that the one routine can be used for a range of different source deduction problems.
- (v) The standard input form for the transport coefficients uses a spectral representation in both space coordinates as well as in time. The convention used in the inputs is to define

$$\Phi(p,y,t) = \sum_k \sum_l \sum_m \phi_{klm} g_k(p) g_l[(y+1)/2] g_m(2t)$$

where

$$g_n(x) = \cos(n\pi x), n \geq 0$$

$$= \sin(n\pi x), n < 0.$$

The reason for this choice is that by defining symmetry properties for the fields in a manner consistent with the boundary conditions some of the series reduce to only sine or cosine series and by ensuring the maximum possible degree of continuity at the boundary, the convergence is improved. The main reason for choosing a spectral representation is to aid the implementation of the model in a manner that is independent of the resolution. The input routines simply take the spectral expansion and evaluate the Fourier series at the (p,y) values at which the fields are needed. The only exception occurs for the fields  $K_{yy}$  and  $K_{pp}$  where spurious 'wiggles' associated with poor convergence can lead to negative values that cause an instability in the transports. It is necessary to set a minimum value for these fields and use this minimum to replace any smaller values that arise from the series. The  $\psi$  and  $K_{py}$  series are zero at all boundaries and by using only sine series, these boundary conditions are satisfied automatically. Implicitly this is defining a double range for each coordinate (reflecting about the zero of p and  $\frac{1}{2}(y+1)$ ) and taking  $\psi$  and  $K_{py}$  as odd functions of each variable. This definition also ensures that the first derivatives of these fields will be continuous at these boundaries. The functions  $K_{yy}$  and  $K_{pp}$  are regarded as even functions under the same reflections so that only cosines of the spatial variables occur. The field  $K_{pp} \rightarrow 0$  as  $p^2$  when  $p \rightarrow 0$ . If the series is written as



$$\sum a_{klm} [g_k(p) - g_{k+1}(p)] g_i [\frac{1}{2}(y+1)] g_m (2t)$$

then this condition is satisfied. If an even function is assumed for  $K_{pp}$  then at  $p = 0$  the first and second derivatives are continuous. The field  $K_{yy}$  is required to vanish quadratically at  $y = \pm 1$  so that the appropriate re-expression of the series is

$$\sum a_{klm} \{g_i [\frac{1}{2}(y+1)] - g_{i+2} [\frac{1}{2}(y+1)]\} g_k(p) g_m (2t).$$

The lists of coefficients for  $K_{yy}$  and  $K_{pp}$  in Appendix B convert these special cases into the general form (at the expense of repeating some index sets) so that if these lists are truncated after any even number of terms the boundary conditions will be satisfied.

The series shown in the appendix are the dominant contributions which approximate the transport coefficients used by Hyson et al. (1980). The stream functions are from wind-field data compiled by Oort and Rasmussen (1971) and the diffusion coefficients were originally obtained by Hidalgo and Crutzen (1977) but were rescaled by Hyson et al. in order to improve the agreement with distribution of  $CCl_3F$ . It should also be noted that the fields  $K_{yy}$ ,  $K_{pp}$  etc. are not the same fields as used by Hyson et al. (1980). In particular Hyson et al. used  $K_{yy}$  to denote the field that was called  $K_{\phi\phi}$  by Enting (1984). In addition the units have been changed so as to express the present fields in a 'dimensionless' form. The transformations connecting the present fields and those tabulated by Hyson et al. (1980) are given by Enting (1984). An additional set of stream function data based on the work of Oort (1983) is also listed. One of the tests presented in Section 5 uses this set as an alternative to the older Oort and Rasmussen data used by Hyson et al. The results are found to be similar in each case.

## 5. Examples

This section presents the results of some trial calculations as an illustration of the type of results that can be expected in practice. Full details of the application of these techniques to specific problems will be presented elsewhere. The examples that are presented here are those that are most relevant to the problem of analysing the seasonal variation of  $CO_2$ . In most examples the approximate concentration function is defined by

$$C(y,t) = [2.5 - 3.5 \cos(\pi y^*)] \sin 2\pi t \quad \text{ppmv} \quad (5.1a)$$

$$y^* = \frac{1}{2}(y+1) \quad (5.1b)$$

$$y = \text{sine (latitude)} \quad (5.1c)$$

The transport fields were defined by the standard series given in Appendix B and in most cases the model resolution was 8 pressure levels by 10 zones.

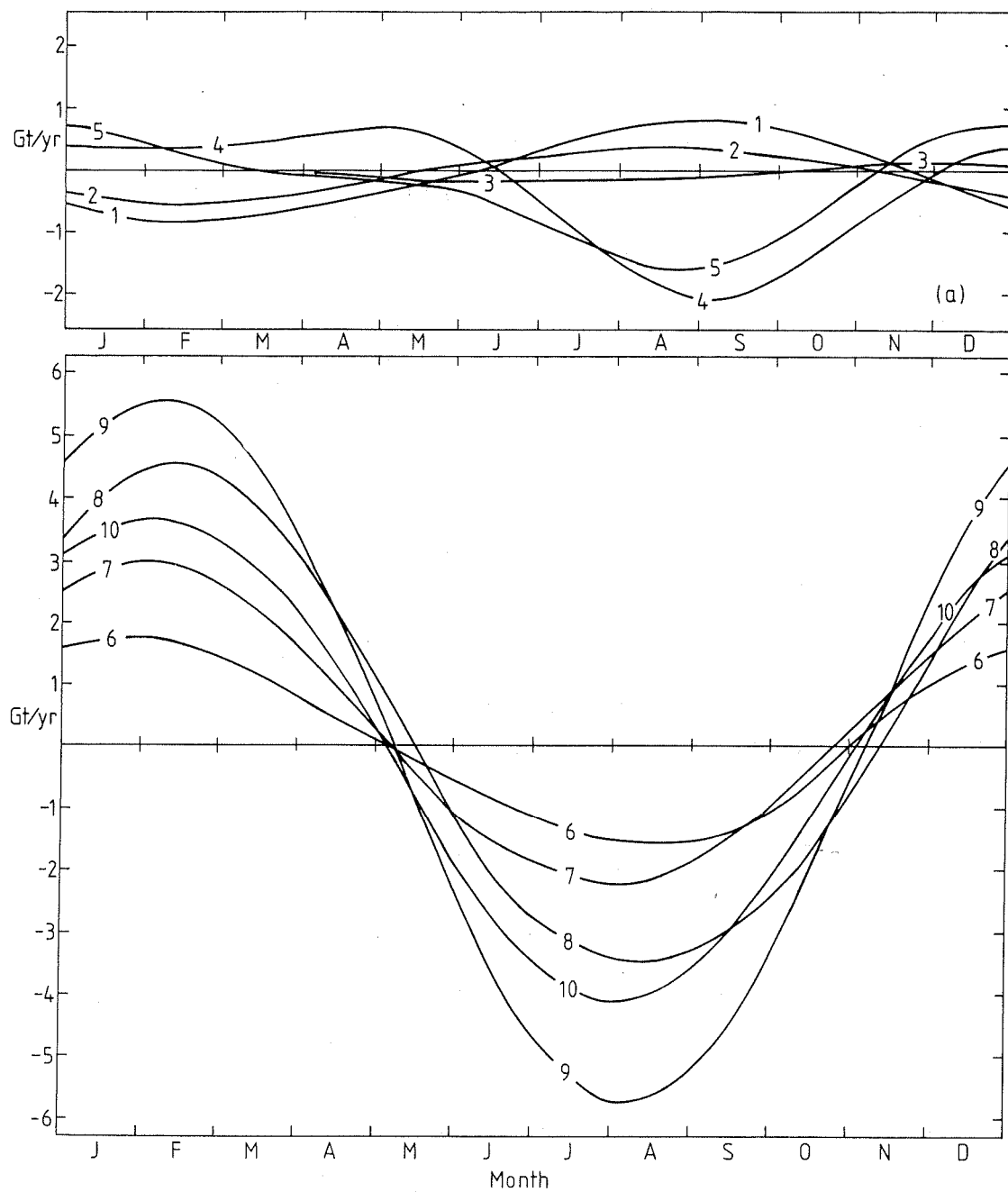


Figure 5.1: Source strengths as functions of time throughout the year deduced from the surface concentration function given by equation (5.1a) for 10 zones, (a) Southern hemisphere zones, 1 (south most) to 5 (tropical); (b) Northern hemisphere zones 6 (tropical) to 10 (north most).

(i) Standard case

This example uses:

- (a) release 5.1a;
- (b) time step 0.002 years;
- (c) Oort and Rasmussen data for stream functions (see Appendix B);
- (d) diffusion coefficients from Hyson et al. (1980), represented by the spectral form from Appendix B and constrained so that

$$K_{yy}/\Delta y^2, K_{pp}/\Delta p^2 \geq 5.0 \quad (5.2)$$

for all points used in the finite difference scheme;

- (e) 8 pressure levels and 10 zones.

The routine DEDUCE based on the code shown in Section 3 was integrated for a total of three years starting from an all zero initial concentration state. The source strengths for each zone were found to have converged to within 0.01 Gt/yr after 1.5 years. Figure 5.1a shows the southern hemisphere sources and Figures 5.1b shows the northern hemisphere sources, taken from the output for the period 2 to 3 years.

It is not possible to draw many conclusions from this case in isolation because it is based on a rough approximation to the actual variation. It should be noted however that in the northern zones, the seasonal CO<sub>2</sub> releases are comparable to the releases deduced by Pearman and Hyson (1980) and are significantly less than those obtained by Fung et al. (1983). This series is one check on the present model implementation, indicating that the vertical mixing is being treated in a manner consistent with the original model of Hyson et al. (1980). One interesting point that does appear from the curves in Figures 5.1a,b is that the annual mean release is non-zero in various zones. On an annual average there is a net transport of CO<sub>2</sub> from the northern zones to the tropics, in spite of the fact that the annual average gradient is zero at all points for release 5.1a. This arises because of covariance effects between the seasonal variations in the transport and the seasonal variations in the concentrations. In the absence of an annual average transport, the covariance effects lead to a mean interhemispheric gradient; in the absence of such a gradient the covariance effects imply a mean transport.

(ii) Interhemispheric gradient

In order to explore these covariance effects further, the second case considered is that of a fixed interhemispheric gradient with no seasonality.

The concentration is fixed to

$$C(y,t) = 2.5 \cos(\pi y^*) \quad (5.3)$$

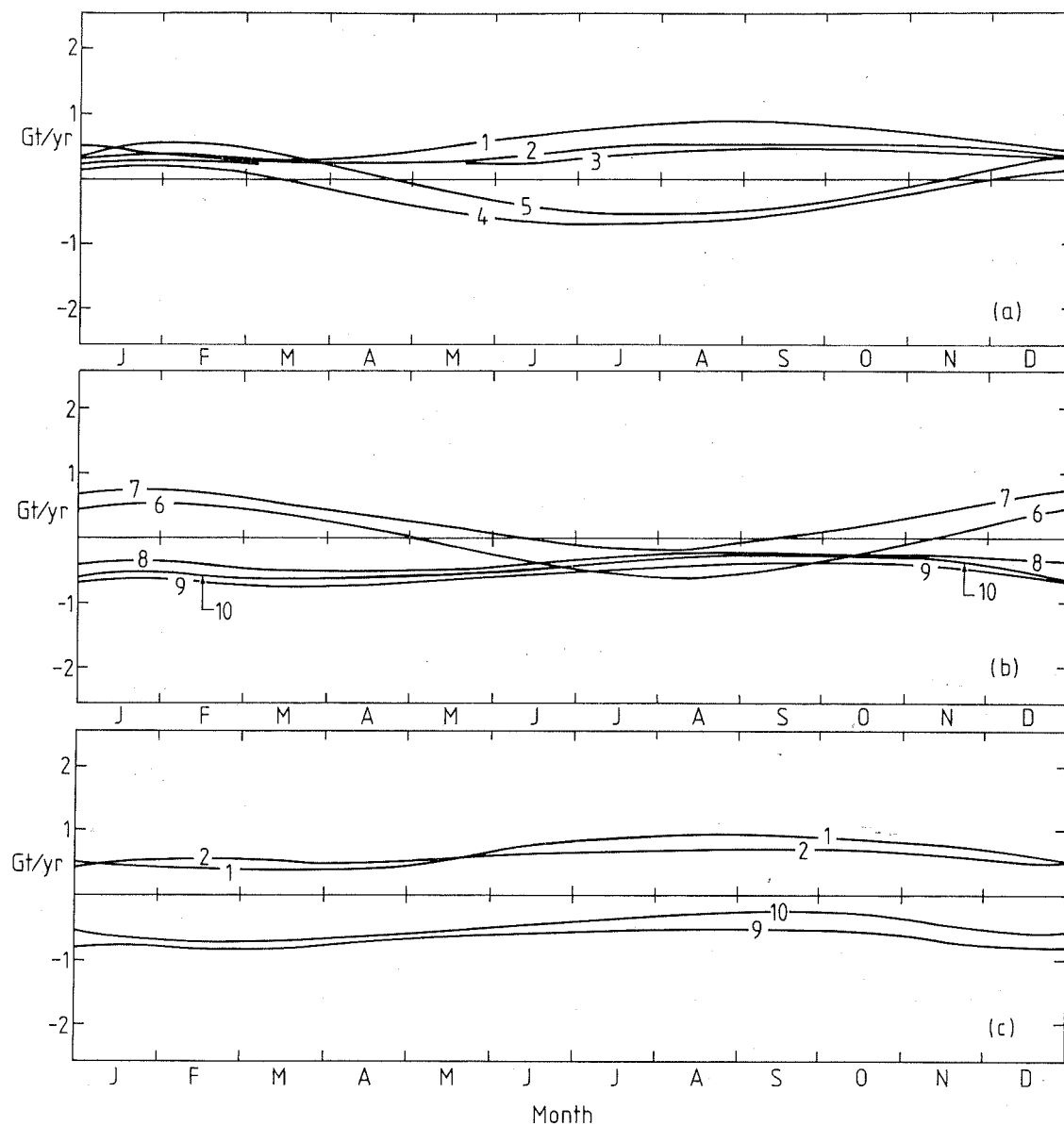


Figure 5.2: Source strengths corresponding to a concentration that is constant in time and varies with latitude according to equation (5.3a). There are relatively strong seasonal variations required in the tropics in order to maintain the maximum gradient in a fixed position while the inter-tropical convergence moves. (a) Southern zones as in Figure 5.1; (b) Northern zones; (c) Sources in zones 1,2,9,10 assuming the corresponding concentrations are fixed and all intermediate sources are zero.

The other conditions are the same as for case (i). The results are shown in Figures (5.2a,b) for northern and southern hemispheres. Due to the covariance effects a strong seasonal source in the tropics is needed to maintain a fixed spatial distribution as the intertropical convergence moves.

(iii) 'Pole-to-Pole' gradient

An alternative to example (ii) that is easier to interpret is to only specify the concentrations at the near polar regions and deduce the transports assuming that all immediate sources are zero. In other words for the 10 zone case, the set  $\alpha$  is restricted to the surface layers of zones 1,2,9 and 10. At these grid points the concentrations are given by (5.3).

The results are shown in Figure 5.2c and it will be seen that even in this case there is a significant seasonal component, varying by  $\pm 30\%$  about the respective means. Thus even on a pole-to-pole basis there is still a significant seasonal component to the transport.

(iv) Change of stream functions

As the first of a series of tests of the sensitivity of the calculations the stream functions derived from the work of Oort and Rasmussen (1971) were replaced by stream functions derived from Oort (1983). (The actual data sets, i.e. the spectral coefficients, are listed in Appendix B). This resulted mainly in small changes to the source strengths that were derived although in the northern hemisphere near the peaks of the release the rates differed by as much as 0.2 Gt/yr.

(v) Change of  $K_{pp}$

In this run the  $K_{pp}$  was multiplied by 1.5. In order to ensure stability the time step had<sup>pp</sup> to be reduced and a step of 0.001 years was used. Apart from these changes, all details were the same as for the standard case (i) above. The changes to southern hemisphere releases were relatively small but the rates of the northern hemisphere releases (and uptakes) increased by as much as 1 Gt/yr near the peaks.

(vi) Change of  $K_{yy}$

In a test similar to case (v) all the  $K_{yy}$  values were multiplied by 1.5 (and again the time step of 0.001 years was<sup>yy</sup> used). This change tended to affect the source strengths in both hemispheres as might be expected. The changes were up to 0.2 Gt/yr.

(vii) Higher frequency

In order to test the sensitivity of the results to some of the less regular variations in the seasonal cycle a release rate of

$$C(y,t) = [1.25 - 1.75 \cos(\pi y^*)] \sin 4\pi t \quad (5.4)$$

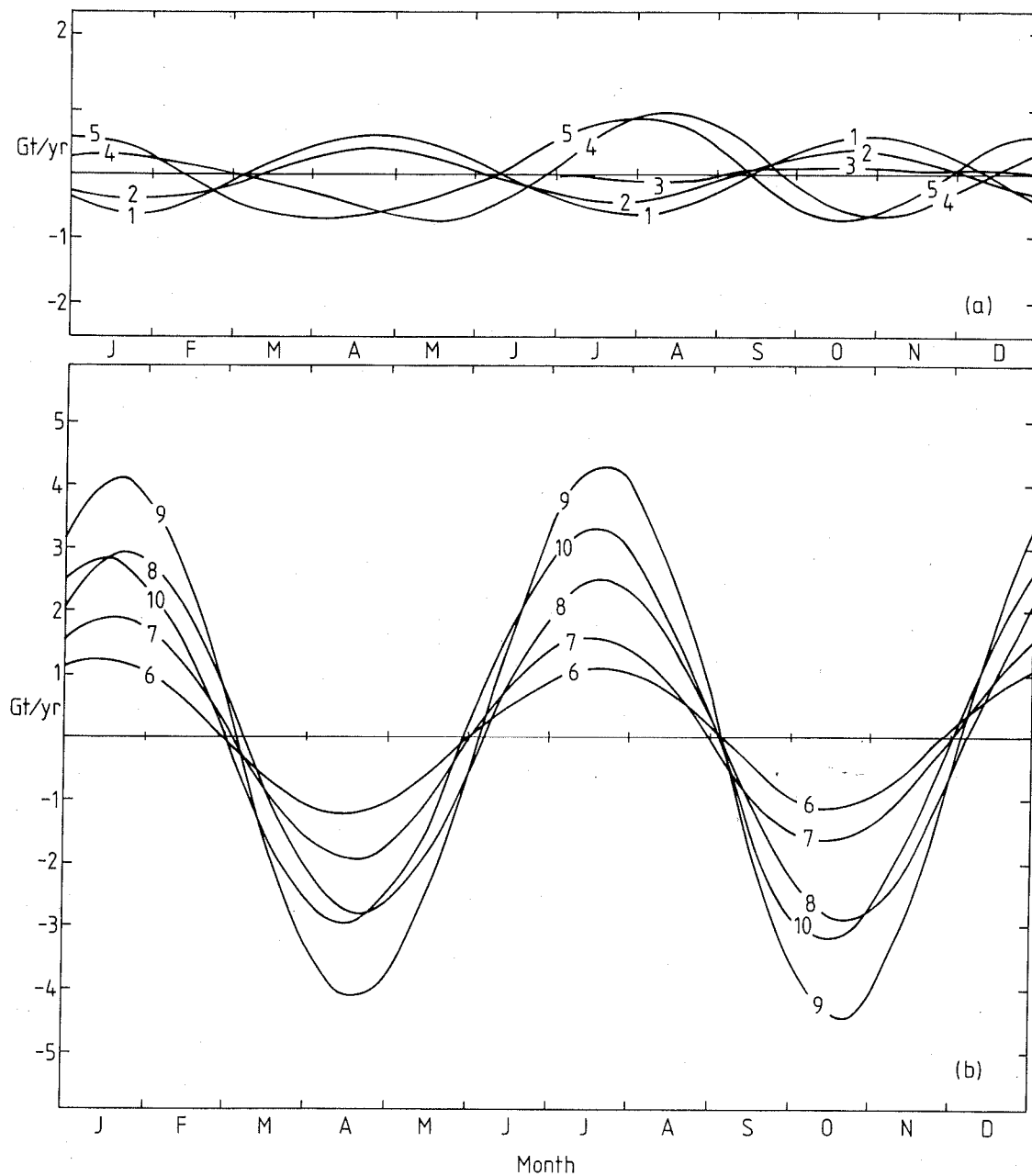


Figure 5.3: Source strengths for release (5.4) with period 6 months  
 (a) Southern zones; (b) Northern zones.

was used. The amplitude was half that of the standard case so that these two cases would have the same amplitude for the time derivatives and so the degree of attenuation of source signals by the transport terms could be compared directly with that in case (i).

The results are shown in Figures (5.3a,b) and they suggest a relatively low damping of higher frequencies indicating that the transport terms play a relatively small role and that a large part of the seasonal signal is due to local sources.

#### (viii) Isolated perturbation

In order to perform a more direct test of the sensitivity of the sources to the observations in the concentrations we consider an isolated error term defined by

$$\left. \begin{aligned} C(y,t) &= 0.5 \cos(2\pi t) && \text{for zone } i \\ &= 0 && \text{otherwise} \end{aligned} \right\} \quad (5.5)$$

As an example Figure (5.4) shows the source strengths corresponding to an isolated perturbation in zone 7.

Again the results indicate a relatively localised influence of the errors in concentrations. It is only the large scale features that combine with the transports to influence the global-scale distributions.

#### (ix) Changing resolution

The final numerical test of the technique involved taking the standard source (5.1a) and using a horizontal resolution of 20 zones. The time step was reduced to 0.001 years to ensure stability. It was found that the combined source strengths for pairs of zones in the 20 zone case agreed very well with the corresponding strengths in the 10 zone case.

### 6. Conclusions

The results presented in the previous sections have shown that the computational techniques described in Section 3 provide a useful method for solving many of the 'source-deduction' problems that arise in connection with the analysis of seasonal variations of CO<sub>2</sub>. The technique provides two major improvements over the approach used by Pearman and Hyson (1980). Firstly it gives a direct solution rather than having to rely on ad hoc fitting procedures and secondly it allows a separation of the atmospheric modelling from the ocean modelling so that a wider range of combinations of models can be considered.

As mentioned in the introduction there are a number of other source deduction problems that have the same mathematical structure, notably the deduction of surface sources of methane given surface concentrations and rate coefficients for the atmospheric sink. The analysis of Section 2 was used to define sufficient conditions for the existence of the unique solutions that can be found using the computational techniques from Section 3. There will however be more general (e.g. non-linear) problems for which the techniques of Section 3 will give a

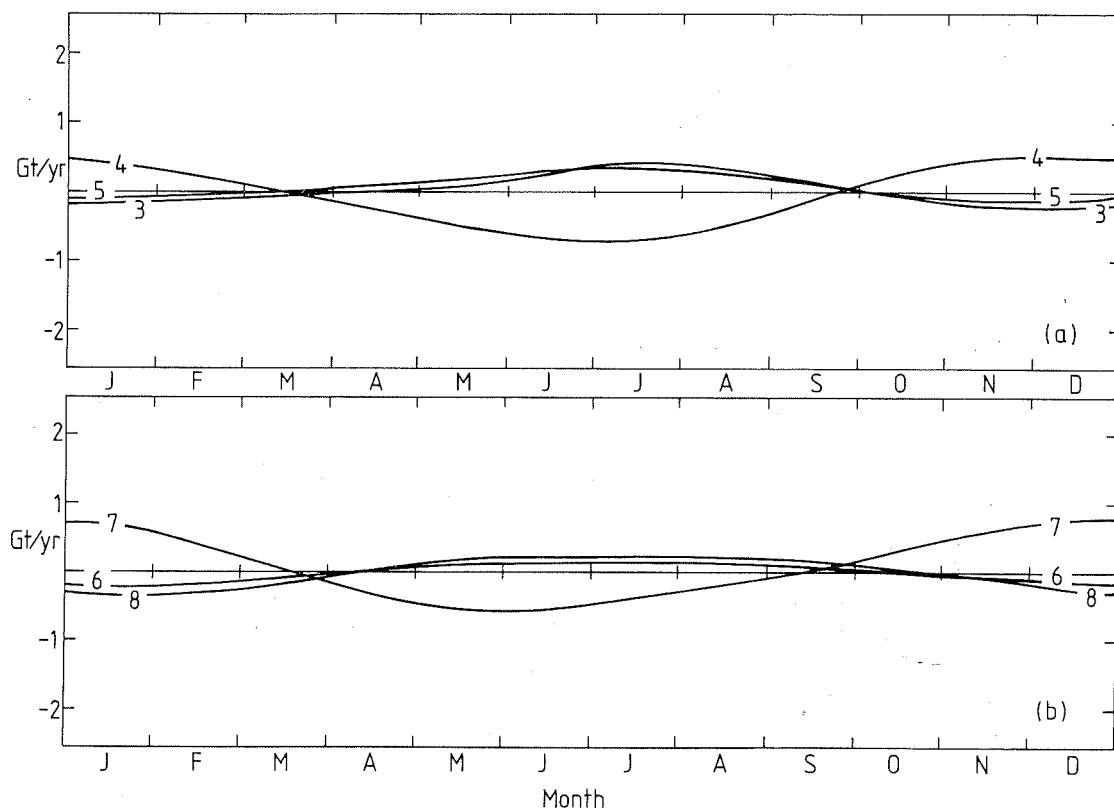


Figure 5.4: Source strengths due to isolated perturbation described by (5.5), with the non-zero concentration occurring in zone 7.



solution even though the proofs of existence and uniqueness are lacking. Of course for non-linear problems it is not possible to subtract off the constant part of the concentration field as was done in the examples in Section 5.

The main examples have concentrated on cases when the grid points are divided into surface points and others so that the surface sources were to be deduced given the concentrations at all points on the surface. However, as indicated by the general formalism of Section 3 and by example (iii) of Section 5, other divisions of sites can be used equally well if the appropriate information is available.

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Appendix A : Green's function formalisms for conserved tracers

Conserved tracers are those for which the total amount of tracer remains constant when there are no sources, i.e.

$$\sum_i \frac{\partial}{\partial t} m_i C_i = \sum_{ij} m_i A_{ij} C_j = 0 \quad (\text{A.1a})$$

for all distributions  $C_j$

$$\text{i.e.} \quad \sum_i m_i A_{ij} = 0 \quad \text{for all } j \quad (\text{A.1b})$$

where the  $m_i$  are the atmospheric masses associated with grid point  $i$ , assuming that the  $C_i$  are in concentration units.

The set of equations is reduced by selecting an arbitrary grid point  $N$  and applying the constraint

$$m_N C_N = - \sum_j' m_j C_j \quad (\text{A.2})$$

where the  $\sum'$  sums exclude grid point  $N$ .

For  $i \neq N$

$$\frac{\partial}{\partial t} C_i = \sum_j A_{ij} C_j + S_i = \sum_j \left( A_{ij} - A_{iN} m_j / m_N \right) C_j + S_i \quad (\text{A.3})$$

This set of equations, together with the constraint (A.2) is only equivalent to the full set of equations if

$$\frac{\partial}{\partial t} \left( \sum m_i C_i \right) = \sum_{ij} m_i A_{ij} C_i + \sum m_i S_i \quad (\text{A.4})$$

$$\text{i.e.} \quad \sum m_i S_i = 0 \quad (\text{A.5})$$

Thus if the constraint (A.2) holds at any initial time and if equation (A.5) holds then the reduced set of equations (A.3) gives the same solutions as the full set of equations. However, under these conditions, the full set of equations has no non-zero solutions when the sources are zero because the 'constant concentration solution is excluded by the constraint and so the reduced set of equations can have no non-zero solutions and a Green's function will exist for the reduced equations.

Appendix B : Sets of transport coefficients used in this study

Each set of transport coefficients is expressed in spectral form as a sum of terms of the form:

$$C_{ijk} g_i(p) g_j(y^*) g_k(2t) \text{ where}$$

$$\begin{aligned} p &= \text{reduced pressure,} && (\text{range } 0 \text{ to } 1) \\ y^* &= \frac{1}{2}(1 + \sin(\text{latitude})), && (\text{range } 0 \text{ to } 1) \\ t &= \text{time in years} \end{aligned}$$

$$\begin{aligned} \text{and } g_n(x) &= \cos(n\pi x), \quad n \geq 0 \\ &= \sin(-n\pi x), \quad n < 0 \end{aligned}$$

The tables list sets of  $i, j, k, C_{ijk}$  in order of decreasing magnitudes of  $C_{ijk}$  so that a natural reduction in resolution of the transport fields can be achieved by taking some smaller number of terms from the beginning of the lists. The fields  $K_{pp}, K_{yy}$  are slightly different - the coefficients are still in the same form but are given in pairs so that truncating these sums after any even number of terms gives a sum that satisfies the boundary conditions described in Section 4. (The boundary conditions of  $\psi$  and  $K_{py}$  are satisfied automatically by each term because sine series are used).

(i) Stream functions from Oort and Rasmussen (1971) 100 terms:

-1	-1	1	.46598E+00	-1	-1	3	-.17645E-01
-1	-3	1	-.29219E+00	-2	-7	0	-.17464E-01
-1	-3	-1	-.25182E+00	-2	-5	-1	-.17375E-01
-1	-4	0	.24666E+00	-1	-7	0	-.17138E-01
-1	-1	-1	.19746E+00	-1	-6	-3	.17136E-01
-1	-5	-1	.16083E+00	-2	-7	1	-.17101E-01
-1	-6	0	-.14045E+00	-7	-5	0	.17067E-01
-1	-5	1	.91587E-01	-4	-1	1	-.16934E-01
-2	-4	0	-.82463E-01	-7	-1	-1	.16324E-01
-1	-10	0	-.72202E-01	-2	-1	-3	-.16263E-01
-2	-3	1	.56330E-01	-2	-2	4	-.16168E-01
-5	-1	1	-.53881E-01	-5	-4	1	-.15922E-01
-2	-1	1	-.50295E-01	-1	-4	-2	-.15906E-01
-1	-2	-2	.48426E-01	-5	-8	1	-.15899E-01
-1	-2	2	.44249E-01	-1	-6	4	.15560E-01
-5	-3	-1	.43824E-01	-1	-4	-3	-.15389E-01
-2	-1	0	.41134E-01	-7	-4	0	-.15254E-01
-5	-1	-1	-.40581E-01	-4	-3	-1	.15003E-01
-5	-5	1	-.40240E-01	-1	-1	-4	-.14881E-01
-1	-2	0	-.38618E-01	-5	-1	3	.14774E-01
-2	-3	-1	.38288E-01	-7	-7	0	-.14723E-01
-1	-9	1	.37255E-01	-2	-5	1	-.14505E-01
-2	-3	0	-.36101E-01	-1	-8	-3	-.14159E-01
-7	-7	1	-.33860E-01	-7	-9	1	.13867E-01
-1	-1	-3	-.33743E-01	-4	-5	1	-.13717E-01
-5	-3	1	.33376E-01	-1	-2	1	.13108E-01
-7	-5	1	.32992E-01	-1	-3	-4	.13099E-01
-1	-7	3	.29563E-01	-4	-1	-1	-.12891E-01
-2	-5	0	.27500E-01	-1	-5	0	.12707E-01
-1	-4	4	-.27261E-01	-1	-9	-3	.12236E-01
-2	-10	0	.27097E-01	-6	-1	0	-.12135E-01
-1	-5	-3	-.24839E-01	-7	-10	0	-.11994E-01
-1	-6	1	.23968E-01	-7	-7	-1	-.11860E-01
-1	-4	-4	.23791E-01	-6	-7	1	.11805E-01
-5	-5	-1	-.22541E-01	-2	-5	3	.11801E-01
-1	-4	1	-.22000E-01	-6	-3	0	.11754E-01
-2	-8	0	.21281E-01	-7	-9	0	.11722E-01
-5	-4	0	-.21183E-01	-1	-8	4	.11689E-01
-1	-2	-4	-.20965E-01	-1	-10	-4	.11683E-01
-1	-8	-2	-.20897E-01	-7	-5	-1	.11504E-01
-5	-7	1	.20443E-01	-6	-5	1	-.11480E-01
-1	-5	3	-.20284E-01	-7	-4	2	-.11439E-01
-7	-1	0	.19419E-01	-1	-7	1	.11434E-01
-5	-6	0	.19175E-01	-1	-4	2	-.11364E-01
-1	-9	0	.18940E-01	-2	-2	2	.11179E-01
-1	-8	1	-.18882E-01	-1	-1	-2	.11161E-01
-7	-3	0	-.18654E-01	-6	-5	0	-.10953E-01
-7	-3	-1	-.18344E-01	-2	-8	-2	-.10710E-01
-5	-6	1	.18196E-01	-5	-1	4	.10566E-01
-5	-10	0	.17649E-01	-1	-6	2	-.10556E-01

(ii) Stream functions from Oort (1983) 100 terms:

-1	-1	1	.43066E+00	-1	-8	2	-.14418E-01
-1	-3	1	-.36390E+00	-5	-3	-1	.14250E-01
-1	-4	0	.22849E+00	-7	-1	1	-.14208E-01
-1	-1	-1	.22198E+00	-1	-11	0	.14098E-01
-1	-3	-1	-.19441E+00	-4	-5	-1	-.14090E-01
-1	-5	-1	.18547E+00	-1	-9	1	-.13910E-01
-1	-5	-1	.11320E+00	-2	-5	-2	-.13829E-01
-1	-6	0	-.95053E-01	-3	-7	1	.13232E-01
-1	-2	0	-.84513E-01	-3	-3	0	.12700E-01
-2	-4	0	-.59400E-01	-7	-3	1	.12586E-01
-4	-1	1	-.50620E-01	-2	-8	0	.12349E-01
-4	-3	1	.46165E-01	-1	-7	-4	-.12022E-01
-5	-1	1	-.40338E-01	-1	-19	1	-.11638E-01
-1	-6	-1	-.36588E-01	-1	-1	-5	.11262E-01
-1	-7	0	.36112E-01	-4	-2	0	.11025E-01
-4	-4	0	-.35767E-01	-1	-1	4	-.10699E-01
-1	-5	-2	.33433E-01	-2	-7	0	-.10491E-01
-1	-1	-3	-.32262E-01	-1	-4	-1	.10428E-01
-1	-7	1	-.32093E-01	-1	-13	0	-.10428E-01
-1	-3	-3	.31170E-01	-1	-20	0	-.10230E-01
-5	-3	1	.31101E-01	-4	-6	0	.10132E-01
-1	-9	0	-.30660E-01	-1	-12	2	.99405E-02
-4	-1	-1	-.28730E-01	-1	-2	1	-.99047E-02
-1	-4	1	.28678E-01	-1	-9	2	.97337E-02
-2	-1	0	.28347E-01	-1	-8	-1	.95149E-02
-1	-7	-1	-.28241E-01	-1	-14	-1	-.94855E-02
-1	-10	0	-.26550E-01	-1	-4	-2	-.94728E-02
-4	-3	-1	.25589E-01	-5	-5	-1	-.93311E-02
-3	-4	0	.24958E-01	-2	-1	1	-.93295E-02
-1	-13	1	-.23635E-01	-1	-18	0	.92994E-02
-2	-3	0	-.22804E-01	-1	-3	4	.92559E-02
-2	-7	1	-.22763E-01	-2	-3	-1	.91002E-02
-2	-3	1	.22752E-01	-1	-5	5	-.89988E-02
-4	-5	1	-.20734E-01	-2	-12	0	.89776E-02
-1	-11	1	.20512E-01	-1	-7	4	-.89250E-02
-5	-5	1	-.19474E-01	-1	-8	3	.89089E-02
-1	-5	-3	-.19159E-01	-5	-7	1	.88972E-02
-5	-1	-1	-.18992E-01	-1	-19	0	-.88908E-02
-1	-3	-2	-.18700E-01	-2	-8	1	.88527E-02
-1	-15	1	.17829E-01	-2	-6	-1	.87543E-02
-1	-6	-2	.17551E-01	-1	-15	-1	.84912E-02
-1	-1	5	-.17375E-01	-7	-4	0	-.84275E-02
-1	-3	5	.16962E-01	-1	-8	-5	-.83245E-02
-2	-6	0	.16214E-01	-3	-4	1	.82965E-02
-1	-5	2	-.16072E-01	-4	-6	-1	.82459E-02
-3	-1	0	-.15990E-01	-1	-9	3	.81531E-02
-1	-8	1	-.15378E-01	-1	-11	2	-.78723E-02
-2	-9	1	.15214E-01	-1	-11	-4	.78436E-02
-1	-16	0	-.15179E-01	-1	-1	-2	.77931E-02
-1	-13	-1	-.14420E-01	-1	-11	4	.77440E-02

(iii)  $K_{yy}$  from Hyson et al. (1980) 88 terms:

0	0	0	.52043E+00	3	4	0	-.88604E-01
0	2	0	-.52043E+00	3	6	0	.88604E-01
0	2	0	.64751E+00	3	1	1	.84337E-01
0	4	0	-.64751E+00	3	3	1	-.84337E-01
0	1	1	-.44722E+00	1	6	0	.83050E-01
0	3	1	.44722E+00	1	8	0	-.83050E-01
0	4	0	.40528E+00	3	6	0	-.76992E-01
0	6	0	-.40528E+00	3	8	0	.76992E-01
0	1	-1	-.34253E+00	4	0	0	-.70522E-01
0	3	-1	.34253E+00	4	2	0	.70522E-01
0	3	1	-.33490E+00	4	2	0	-.91912E-01
0	5	1	.33490E+00	4	4	0	.91912E-01
0	6	0	.30682E+00	3	3	1	.67362E-01
0	8	0	-.30682E+00	3	5	1	-.67362E-01
0	3	-1	-.26468E+00	3	1	-1	.65277E-01
0	5	-1	.26468E+00	3	3	-1	-.65277E-01
0	8	0	.23272E+00	3	8	0	-.65136E-01
0	10	0	-.23272E+00	3	10	0	.65136E-01
1	0	0	.21747E+00	4	4	0	-.61635E-01
1	2	0	-.21747E+00	4	6	0	.61635E-01
1	2	0	.23940E+00	4	1	1	.60776E-01
1	4	0	-.23940E+00	4	3	1	-.60776E-01
0	5	1	-.17872E+00	1	8	0	.55921E-01
0	7	1	.17872E+00	1	10	0	-.55921E-01
1	1	1	-.17681E+00	3	3	-1	.53509E-01
1	3	1	.17681E+00	3	5	-1	-.53509E-01
0	5	-1	-.15138E+00	4	6	0	-.52747E-01
0	7	-1	.15138E+00	4	8	0	.52747E-01
1	1	-1	-.13482E+00	5	2	0	.51074E-01
1	3	-1	.13482E+00	5	4	0	-.51074E-01
1	4	0	.12874E+00	5	4	0	.94121E-01
1	6	0	-.12874E+00	5	6	0	-.94121E-01
0	7	1	-.12652E+00	5	6	0	.12799E+00
0	9	1	.12652E+00	5	8	0	-.12799E+00
1	3	1	-.12100E+00	5	8	0	.13292E+00
1	5	1	.12100E+00	5	10	0	-.13292E+00
0	7	-1	-.11337E+00	1	5	1	-.49400E-01
0	9	-1	.11337E+00	1	7	1	.49400E-01
3	0	0	-.95577E-01	4	3	1	.48464E-01
3	2	0	.95577E-01	4	5	1	-.48464E-01
3	2	0	-.13131E+00	4	1	-1	.47010E-01
3	4	0	.13131E+00	4	3	-1	-.47010E-01
1	3	-1	-.95169E-01	3	5	1	.45877E-01
1	5	-1	.95169E-01	3	7	1	-.45877E-01

(iv)  $K_{yp}$  based on Hyson et al. (1980) 50 terms:

-1	-2	0	.22327E+00
-1	-3	1	-.12942E+00
-1	-4	0	.12114E+00
-1	-3	-1	-.10468E+00
-1	-1	1	-.98400E-01
-2	-2	0	-.96935E-01
-2	-4	0	-.86466E-01
-5	-2	0	-.85467E-01
-2	-3	1	.81963E-01
-5	-4	0	-.80796E-01
-1	-1	-1	-.76309E-01
-5	-3	1	.71358E-01
-2	-5	1	.63491E-01
-2	-3	-1	.53926E-01
-1	-7	1	.50057E-01
-5	-3	-1	.48010E-01
-7	-5	1	-.44883E-01
-7	-6	0	.42705E-01
-2	-1	1	.39980E-01
-1	-6	0	-.39376E-01
-5	-1	1	.39355E-01
-2	-6	0	-.39048E-01
-2	-5	-1	.39016E-01
-5	-5	1	.37917E-01
-7	-7	1	-.33210E-01
-1	-7	-1	.30968E-01
-1	-8	0	-.30740E-01
-4	-2	0	-.29668E-01
-6	-2	0	-.29213E-01
-2	-7	1	.27490E-01
-2	-1	-1	.27094E-01
-5	-1	-1	.26787E-01
-7	-5	-1	-.25570E-01
-5	-5	-1	.25382E-01
-4	-4	0	-.24711E-01
-7	-4	0	.24237E-01
-4	-3	1	.22870E-01
-7	-7	-1	-.22186E-01
-7	-8	0	.21134E-01
-5	-6	0	-.20968E-01
-6	-7	1	-.20535E-01
-5	-7	0	.19546E-01
-7	-3	1	-.19001E-01
-5	-9	1	-.18672E-01
-1	-5	0	.17348E-01
-2	-10	0	.17261E-01
-2	-5	0	.16885E-01
-6	-8	0	.16617E-01
-2	-7	-1	.16398E-01
-6	-9	1	-.15847E-01



(v)  $K_{pp}$  based on Hyson et al. (1980) 86 terms. Note that to this order of approximation there is no seasonal variation.

0	0	0	.12969E+01	1	10	0	.14377E+00
1	0	0	-.12969E+01	3	4	0	-.12631E+00
1	2	0	-.82171E+00	4	4	0	.12631E+00
2	2	0	.82171E+00	3	5	0	-.12564E+00
3	2	0	.79664E+00	4	5	0	.12564E+00
4	2	0	-.79664E+00	1	7	0	-.12525E+00
1	0	0	.77360E+00	2	7	0	.12525E+00
2	0	0	-.77360E+00	2	7	0	-.15291E+00
4	2	0	.52048E+00	3	7	0	.15291E+00
5	2	0	-.52048E+00	0	7	0	.12242E+00
2	2	0	-.31706E+00	1	7	0	-.12242E+00
3	2	0	.31706E+00	1	7	0	.12242E+00
3	2	0	-.31706E+00	2	7	0	-.12242E+00
4	2	0	.31706E+00	2	7	0	.12242E+00
4	2	0	-.31706E+00	3	7	0	-.12242E+00
5	2	0	.31706E+00	3	9	0	.11958E+00
0	5	0	.30477E+00	4	9	0	-.11958E+00
1	5	0	-.30477E+00	0	3	0	-.11053E+00
3	0	0	-.27748E+00	1	3	0	.11053E+00
4	0	0	.27748E+00	1	3	0	-.11053E+00
0	4	0	.25849E+00	2	3	0	.11053E+00
1	4	0	-.25849E+00	2	3	0	-.14315E+00
1	4	0	.29910E+00	3	3	0	.14315E+00
2	4	0	-.29910E+00	3	3	0	-.14315E+00
0	8	0	-.23772E+00	4	3	0	.14315E+00
1	8	0	.23772E+00	4	3	0	-.14315E+00
0	6	0	-.23104E+00	5	3	0	.14315E+00
1	6	0	.23104E+00	2	9	0	.10813E+00
3	3	0	.22483E+00	3	9	0	-.10813E+00
4	3	0	-.22483E+00	3	9	0	.10813E+00
4	0	0	-.22281E+00	4	9	0	-.10813E+00
5	0	0	.22281E+00	4	9	0	.14449E+00
1	5	0	.21825E+00	5	9	0	-.14449E+00
2	5	0	-.21825E+00	4	1	0	-.10453E+00
4	3	0	.18602E+00	5	1	0	.10453E+00
5	3	0	-.18602E+00	2	6	0	.99772E-01
1	3	0	-.18190E+00	3	6	0	-.99772E-01
2	3	0	.18190E+00	3	6	0	.15028E+00
3	1	0	-.17105E+00	4	6	0	-.15028E+00
4	1	0	.17105E+00	4	6	0	.11191E+00
1	1	0	.14775E+00	5	6	0	-.11191E+00
2	1	0	-.14775E+00	2	0	0	.98169E-01
0	10	0	-.14377E+00	3	0	0	-.98169E-01