Movement Up and Down: Modeling Dive Depth of Harbor Seals from Time Depth Recorders

Jay Ver Hoef[1], Megan Higgs[2], and Josh London[1]

[1]NOAA National Marine Mammal Lab
NMFS Alaska Fisheries Science Center
Seattle, Washington, USA
[2]Department of Statistics, Montana State University
Bozeman, Montana, USA

Acknowledgements

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Study maximum diving depth of seals

- We would like a continuous record of dive depth in meters, but...
- Logistical constraints of satellite time-depth-recorders (TDRs)
  - Data storage aboard the mammal
  - Transmission of data to satellite

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- Our main goal: quantify and describe relationships between covariates and the categorical response.
- Also interested in predicting missing data
- Covariates of interest:
  - Time of Day (4 categories)
  - Day of Year
  - Season (Fall, Spring, Pupping)
  - Ocean depth?
  - Sex
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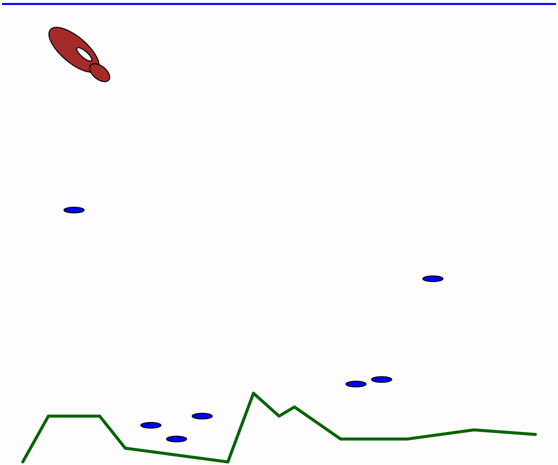

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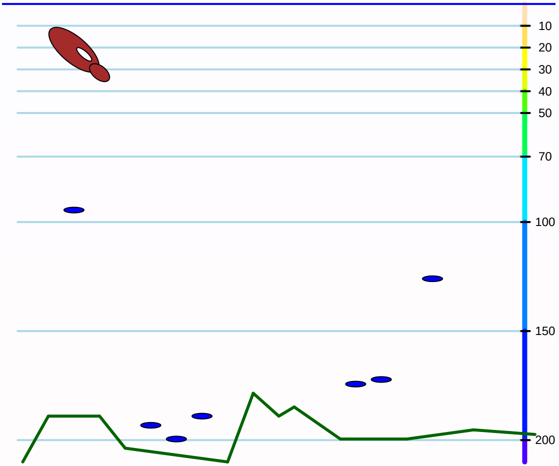

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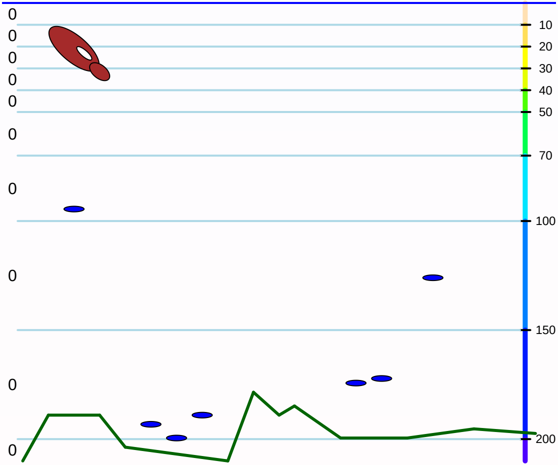

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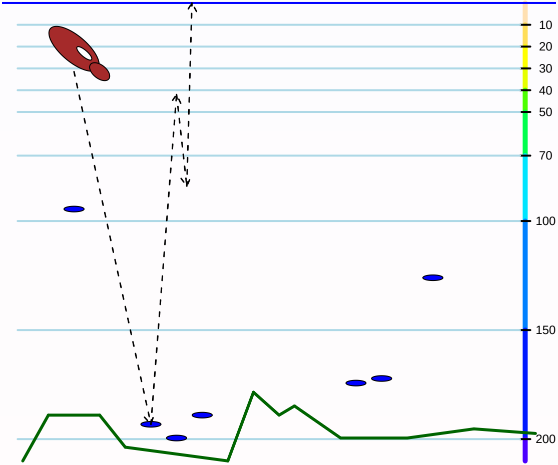

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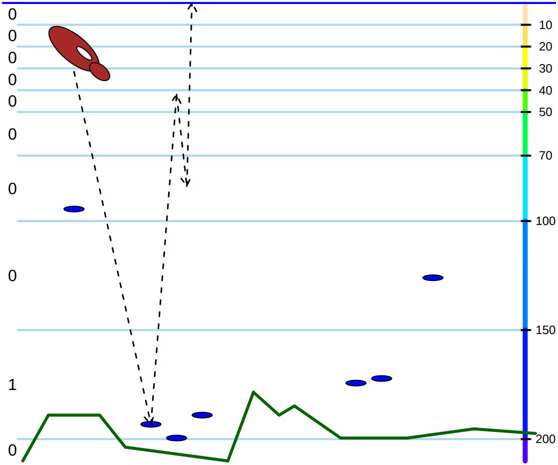

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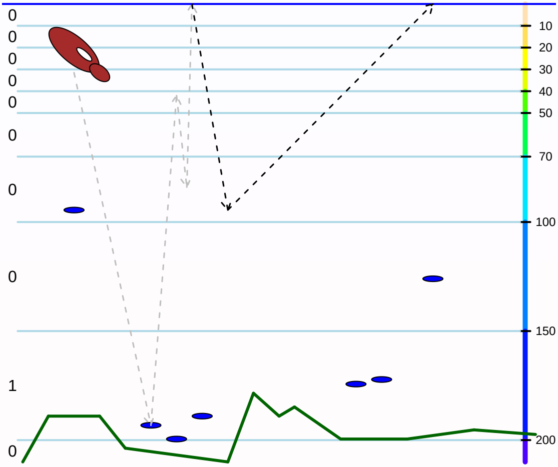

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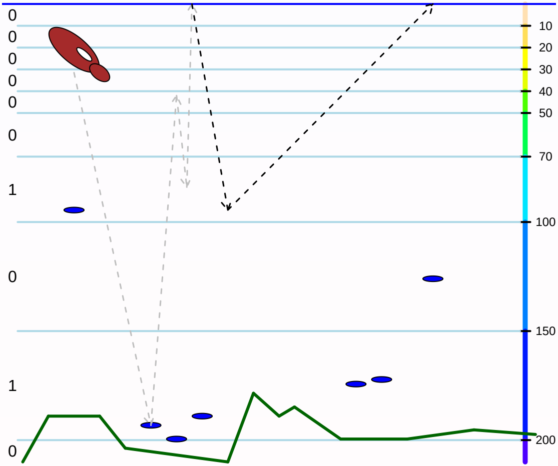

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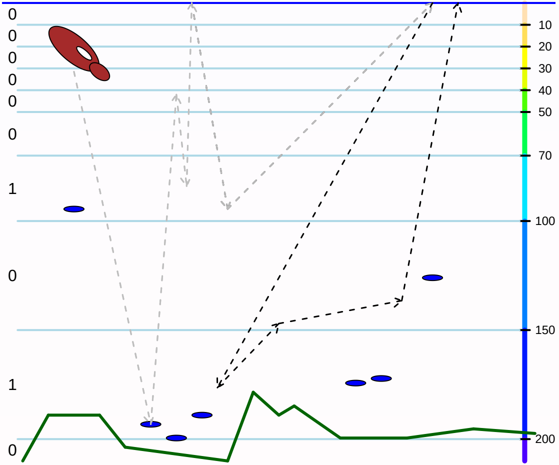

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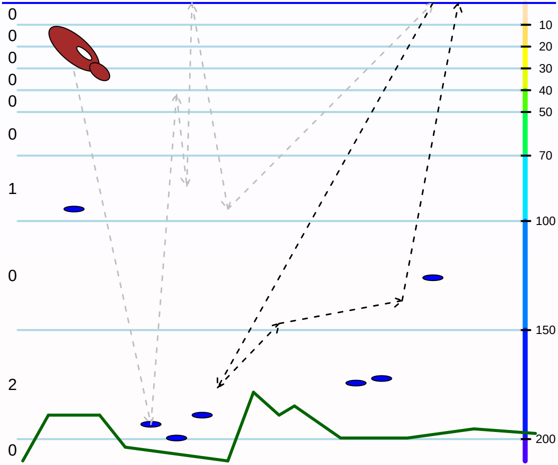

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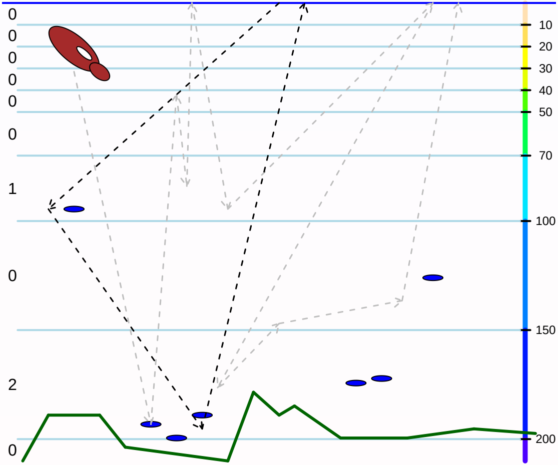

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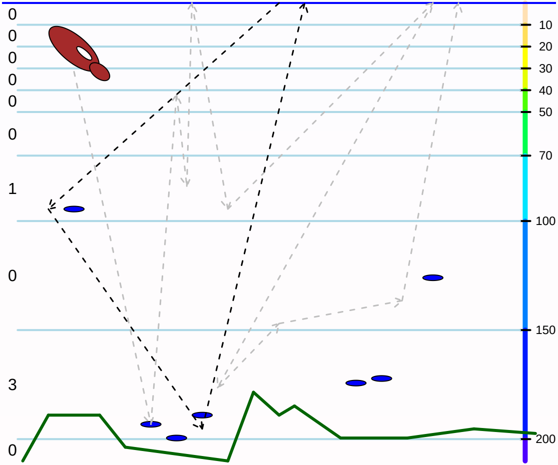

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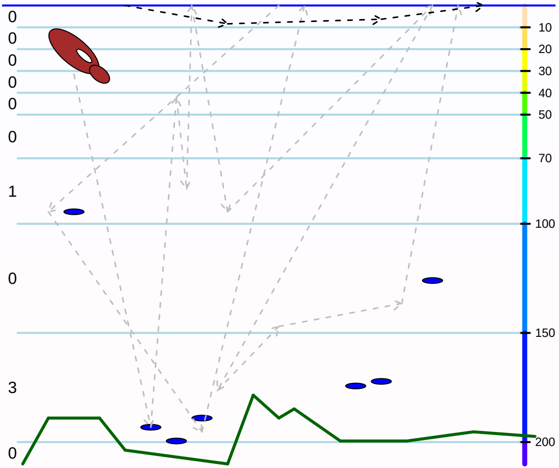

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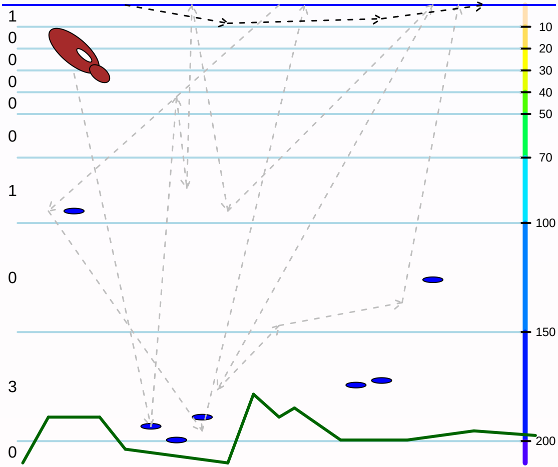

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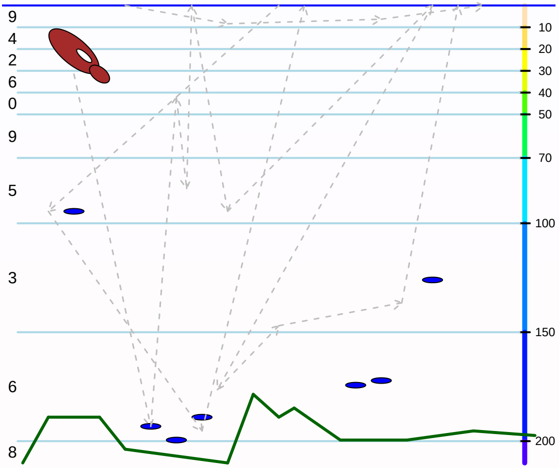

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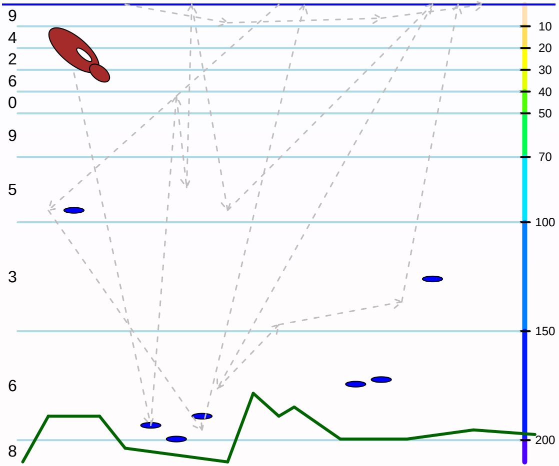
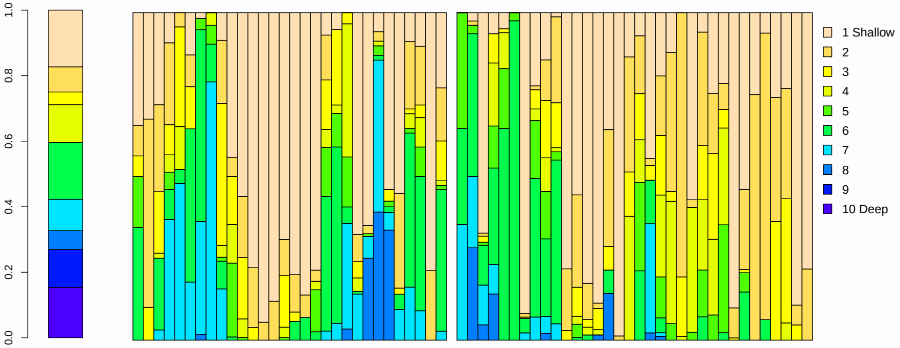


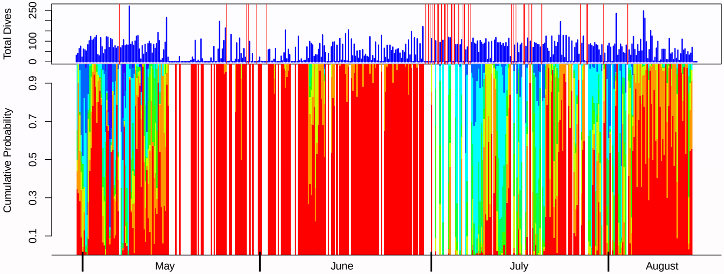
= 52 total dives in 6 hours

Motivating Data



Motivating Data

- Adult Female



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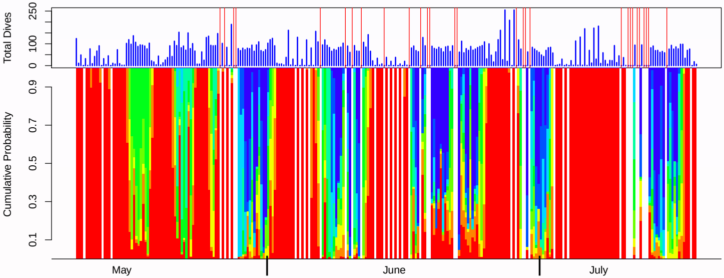

Characteristics of the data

- Continuous dive depth is categorized into *ordered categories* with a practically meaningful set of bin boundaries
  - Ordinal data discretized from continuous behavior
- Aggregated over time (6 hr. intervals) into *multi-category counts*
- *Time series* of multi-category counts for each animal
- Multiple animals and years

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Hierarchical Model

- Total Number of Dives
  - $n_i \sim [(\text{Total Number Dives})_i | \text{covariates}_i] = \text{Poi}(\lambda_i)$
  - $\log(\lambda_i) = \mathbf{x}_i'\beta + \epsilon_i$
  - $\{\epsilon_i\}$ are temporally autocorrelated
- Categorical Counts
  - $\mathbf{y}_i \sim [(\text{Counts per Category})_i | n_i, \text{covariates}_i] = \text{Mult}(n_i, \mathbf{p}_i)$
  - $f(\mathbf{p}_i); \mathbf{x}_i, \delta_{i,k}$?
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  - For unordered categorical data
  - E.g., counties, colors, etc.
- Cumulative Logit Model
  - For ordered categorical data
  - E.g, Strongly Agree $\rightarrow$ Strongly Disagree
- Aggregated Continuous-value Models
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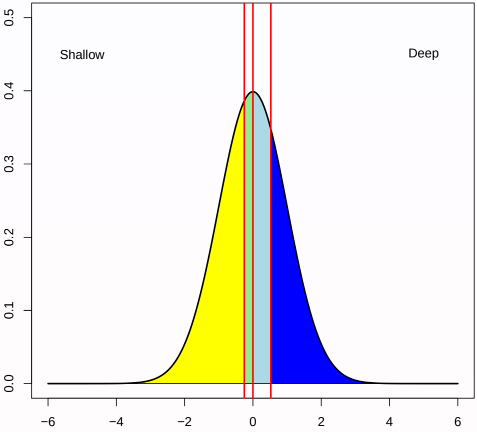
- Create probabilities by cutting a standard normal distribution
- The p_k will be the probability between cutpoints
- Then model the cutpoints with covariates and autocorrelation



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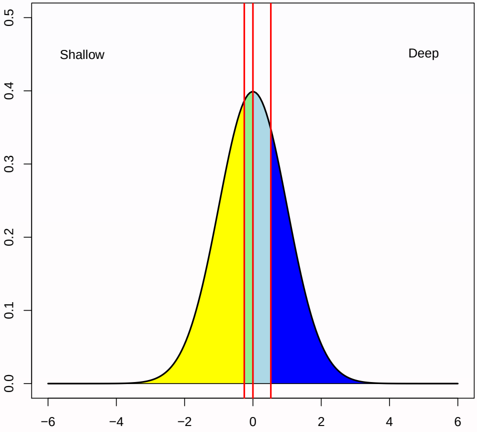

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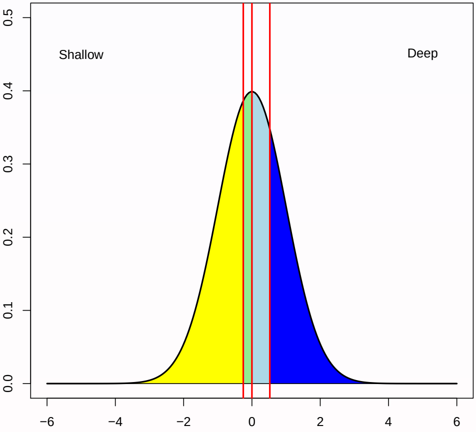

Cutpoint Model

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- To keep order relations, need to model additive increments $\log(a_{k,i}) = \mathbf{x}_i'\boldsymbol{\theta}_k + \delta_{k,i}$
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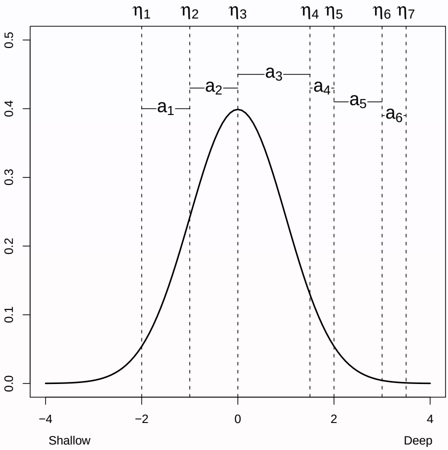

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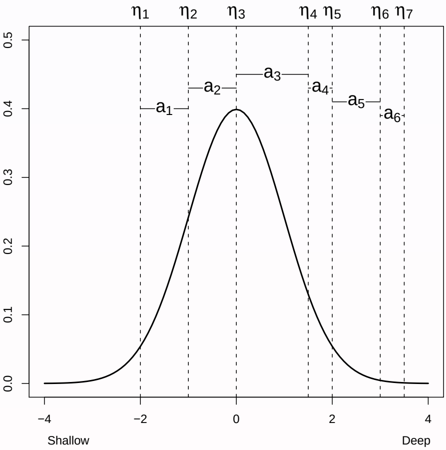

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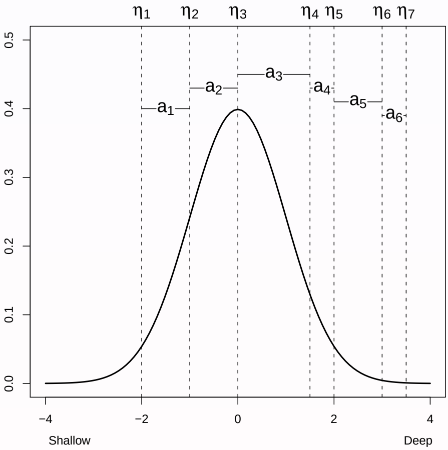

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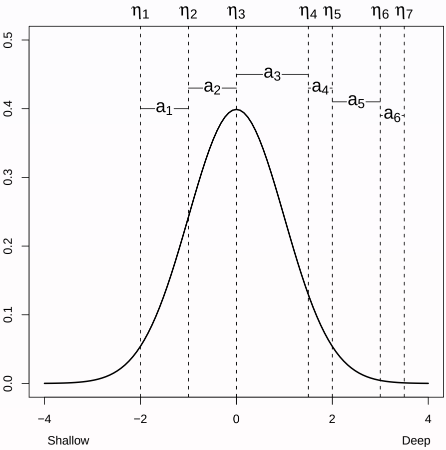

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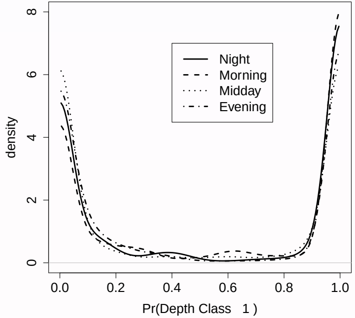
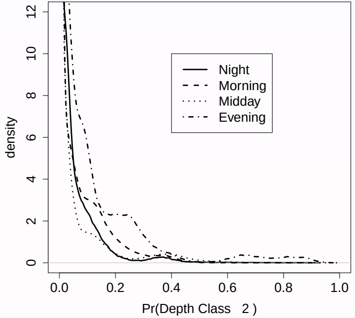
Full Posterior Distribution

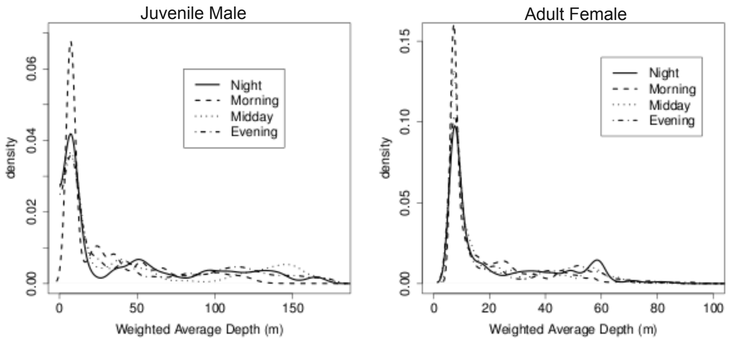
Depth Class



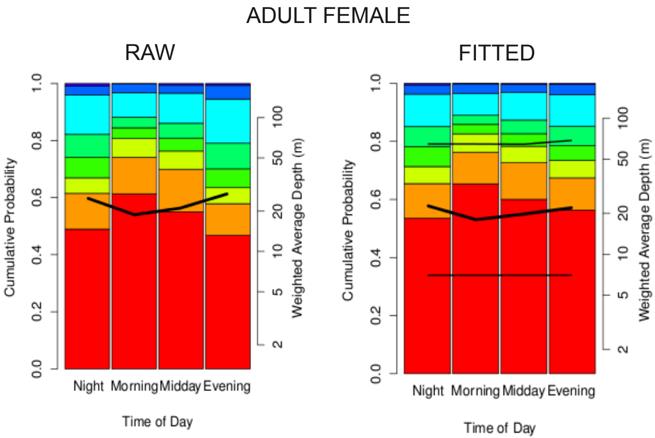


Full Posterior Distribution

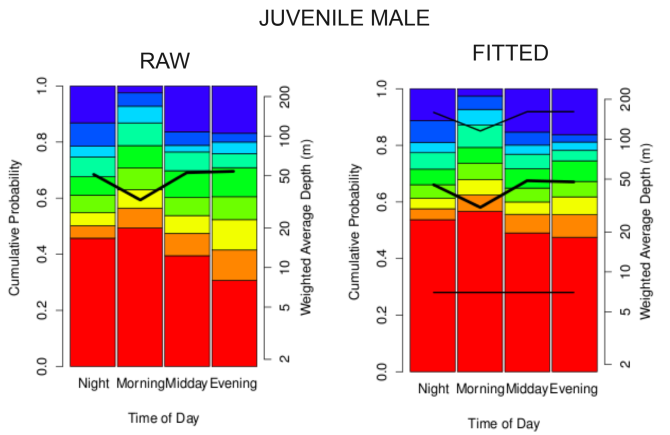
Weighted Average Depth



Time of Day Effect

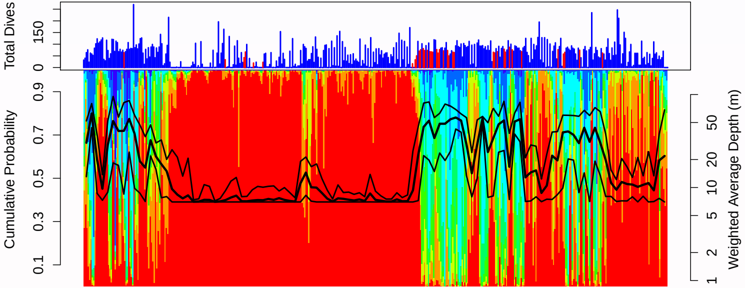


Time of Day Effect



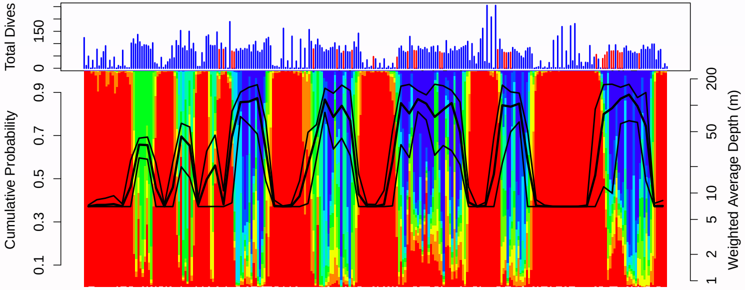
Predictions

Adult Female



Predictions

Juvenile Male



Conclusions

- We can effectively use hierarchical cutpoint models to:
  - model effect of covariates on overall counts and category probabilities,
  - estimate full posterior distributions of category probabilities,
  - compute functions of probabilities (e.g., weighted average depth) using full posterior distribution, and
  - Make predictions for unobserved time periods.
- Manuscript submitted:
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Further Work

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- Models need to be extended to multiple animals.
- Develop an R package?

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